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A Robust and Adaptive Lyapunov-Sliding Mode Control Strategy for Electric Motor Applications

Tidiane Diaby
Department of Electrical and Computer Engineering,
Oakland University, Rochester, MI-48309 USA and
Department of Electrical and Computer Engineering,
University of Michigan, Dearborn, MI-48128 USA

Mohamed Ali Zohdy
Department of Electrical and Computer Engineering,
Oakland University, Rochester, MI-48309 USA and
Department of Electrical and Computer Engineering,
University of Michigan, Dearborn, MI-48128 USA

Adnan Shaout
Department of Electrical and Computer Engineering,
Oakland University, Rochester, MI-48309 USA and
Department of Electrical and Computer Engineering,
University of Michigan, Dearborn, MI-48128 USA

ABSTRACT

This paper presents a robust and adaptive control strategy that integrates Lyapunov-based stability and Discrete Time Sliding Mode Control (DTSMC) for speed tracking, and stability enhancement in electric motors. The proposed approach leverages Lyapunov theory to guarantee system stability by designing a positive definite Lyapunov function, ensuring the convergence of tracking errors under varying operating conditions. Simultaneously, the discrete SMC provides robust disturbance rejection and resilience to parameter variations by maintaining system states on a predefined sliding surface. The combination of these two methods addresses the limitations of conventional controllers, which often lack robustness to uncertainties and Disturbances. The effectiveness of the proposed method is validated through theoretical analysis and simulation results on PMSM model, demonstrating superior tracking performance, under different load conditions and system uncertainties.

Keywords: Lyapunov, Discrete Time Sliding Mode Control (DTSMC), PID, FOC, PMSM, OLB-SMC, quasi-sliding band (QSB).

INTRODUCTION

Electric motors, particularly Permanent Magnet Synchronous Motors (PMSMs), are an integral component in various industrial, automotive, and household applications due to their high efficiency, reliability, and robust performance. However, achieving precise control over these motors, especially under varying operational conditions, remains a significant challenge.

Accurate speed tracking, stability maintenance, and energy efficiency optimization are critical objectives in motor control. These objectives are often compromised by factors such as parameter variations, external disturbances, and inherent nonlinearities of the motor system.
Traditional control strategies, including Proportional-Integral-Derivative (PID) controllers and conventional Field-Oriented Control (FOC), have been widely adopted for motor control. While these methods offer simplicity and ease of implementation, they exhibit several limitations such as sensitivity to parameter variations, lack of robustness, Energy Inefficiency. Conventional controllers require precise knowledge of motor parameters. Variations in stator resistance, rotor resistance, and inductance due to temperature changes or aging can lead to degraded performance [1]. These methods often struggle to maintain desired performance levels in the presence of external disturbances and model uncertainties [2]. Traditional control approaches typically do not incorporate mechanisms for real-time energy optimization, resulting in suboptimal energy consumption [3]. To address these challenges, advanced control techniques have been explored. Sliding Mode Control (SMC) has gained attention for its robustness against system uncertainties and external disturbances [4]. SMC operates by driving the system states onto a predefined sliding surface and maintains them there, ensuring robustness and finite-time convergence. However, SMC is known to induce chatter, a phenomenon characterized by high-frequency oscillations, which can excite unmodeled dynamics and cause wear in mechanical components [5]. Lyapunov-based control methods offer a systematic approach to ensure system stability by constructing a Lyapunov function that decreases over time, guaranteeing convergence to equilibrium [6]. These methods provide theoretical guarantees of stability but may lack robustness when dealing with significant uncertainties or disturbances. Recent research has attempted to combine the strengths of Lyapunov-based stability and SMC to develop robust and stable control strategies. For instance, an optimal Lyapunov-based sliding mode control (OLB-SMC) approach has been proposed to control the movements and rotation of slot-less self-bearing motors, effectively rejecting the effects of uncertainties and disturbances [7]. This paper proposes a robust and adaptive control strategy that Cooperatively combines Lyapunov-based stability and Discrete Time Sliding Mode Control (DTSMC) to enhance speed tracking, and stability, in a PMSM motor. The main contributions of this work are hybrid control strategy, adaptive DTSMC, and comprehensive analysis and validation. In summary, the limitations of conventional motor control strategies, including their sensitivity to parameter variations, lack of robustness, and energy inefficiency, highlight the need for a more advanced approach. The proposed control strategy, which integrates Lyapunov-based methods with the Discrete Time Sliding Mode Control (DTSMC), offers a robust and adaptive solution capable of maintaining precise speed tracking, and ensuring stability under a wide range of operating conditions. The subsequent sections of this paper present a detailed formulation of the proposed control methodology, including the mathematical derivations, design of the Lyapunov-based stability function, and DTSMC components. Comprehensive theoretical analysis and simulation results are also provided to demonstrate the effectiveness and superiority of the proposed approach.

SYSTEM MODEL

In this section, we develop the mathematical model of the Permanent Magnet Synchronous Motor (PMSM) in the dq-reference frame. These models are essential for designing robust control strategies, as they allow the control of motor dynamics in a stationary or rotating reference frame, facilitating the application of vector control methods. Figure1

PMSM Mathematical Model in dq-Reference Frame
Voltage Equations:

$$V_d = R_s i_d + \frac{d\lambda_d}{dt} - \omega_e \lambda_q \quad (1)$$

$$V_q = R_s i_q + \frac{d\lambda_q}{dt} - \omega_e \lambda_d \quad (2)$$

Where:

- V_d, V_q are d-axis and q-axis stator voltages (V)
- i_d, i_q are d-axis and q-axis stator currents (A)
- R_s is the stator resistance (Ω)
- λ_d, λ_q are d-axis and q-axis flux linkages (Wb)
- ω_e is the Electrical angular speed (rad/s)

Flux Linkage Equations:

$$\lambda_d = L_d i_d + \lambda_f \quad (3)$$

$$\lambda_q = L_q i_q \quad (4)$$

Where:

- L_d, L_q are d-axis and q-axis inductances (H)
- λ_f is the Permanent Magnet flux linkage (Wb)

Electromagnetic Torque:

$$T_e = \frac{3}{2} P \left(\lambda_d i_q - \lambda_q i_d \right) \quad (5)$$

Where:

- P = Number of pole pairs

Mechanical Equation:

$$J \frac{d\omega_m}{dt} + B\omega_m = T_e - T_L \quad (6)$$

Where:

- J is the Moment of inertia ($kg.m^2$)

- B is Damping coefficient (N.m.s/rad)
- ω_m is Mechanical angular speed (rad/s)
- T_L is Load torque (N.m)

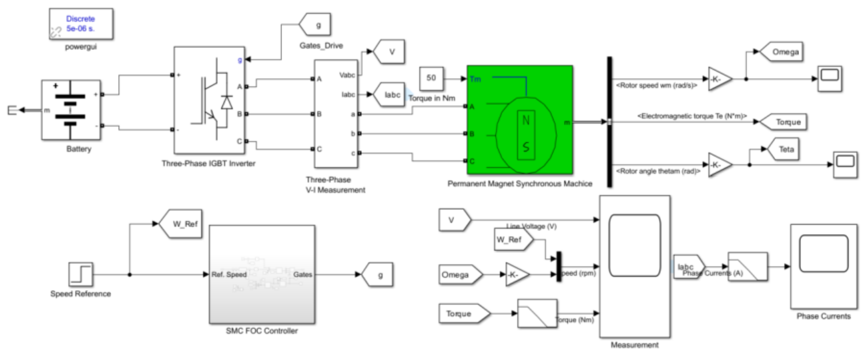

Figure1: PMSM Model

CONTROL DESIGN

In this section, we present the design of a robust and adaptive control strategy that combines Lyapunov-based stability and Discrete Time Sliding Mode Control (DTSMC) for Permanent Magnet Synchronous Motors (PMSM). The proposed hybrid control approach is developed within the framework of Field-Oriented Control (FOC), which provides dynamic decoupling of torque and flux components, enabling precise manipulation of motor performance. By integrating the stability guarantees of Lyapunov theory with robustness and disturbance rejection capabilities of the DTSMC, the controller ensures high-performance speed tracking, enhanced system stability, and resilience to parameter variations and external disturbances, while retaining the real-time effectiveness of the FOC structure. The control objective is to control the rotor speed ω_r to its reference ω_{ref}.

Defining the Sliding Surface and Reaching Law

Sliding surface (also called the sliding manifold) [8] serves as a critical design step in our hybrid control system design. In discrete-time SMC, the sliding surface is defined as a linear combination of the tracking error and its forward difference in (9). The y-axis of the sliding surface Graph in Figure 2 is $S(k)$(sliding variable at sample k). Initially, $S(k)$ decreases towards zero (the reaching phase). After reaching the band (red dashed lines), $S(k)$ oscillates inside it, this is the boundary layer or quasi-sliding mode in discrete time [9]. The oscillations around zero are characteristic of sign-based control in discrete systems, where perfect sliding cannot be maintained due to sampling, so a bounded "band" is maintained. In discrete time sliding mode control (DTSMC), the **reaching law** defines how the sliding variable $S(k)$ evolves at each sampling instant. For our design we used **constant reaching law with boundary layer**, the dynamics are given in (7) and (8)

$$s(k+1) = (1-\eta) - \eta\, sat\left(\frac{s(k)}{\phi}\right) \qquad (7)$$

Where:

- $\eta > 0$ is the reaching gain (or rate), which controls how aggressively the system is driven toward the sliding surface.
- $\phi > 0$ defines the boundary layer thickness, providing a tolerance region around the ideal sliding surface.
- $sat(\frac{s}{\phi})$ is the saturation function, defined as:

$$sat\left(\frac{S}{\phi}\right) = \begin{cases} +1 & S > \phi \\ \frac{S}{\phi} & |S| \leq \phi \\ -1 & S < -\phi \end{cases} \quad (8)$$

Outside the boundary layer ($|S| > \phi$): the control behaves like a classical **sign function**, driving the state forcefully toward the sliding surface. **Inside the boundary layer** ($|S| \leq \phi$): the control is **linearized** by the saturation function. This ensures smoother control action and avoids excessive oscillations (chattering). Thus, the system does not converge exactly to $s = 0$ but instead remains bounded within the quasi–sliding band $[-\phi, +\phi]$. This produces the characteristic **zigzag motion** around the sliding surface seen in discrete-time implementations (Figure_). The width of this zigzag band is directly determined by the choice of ϕ and the sampling period (T_s=5e-6sec). The **reaching law with boundary layer** ensures that the sliding variable converges rapidly and stays within the defined tolerance band. This approach **balances robustness and smoothness in such a way that** Robustness is guaranteed by the discontinuous control action outside the boundary layer and Smoothness is introduced by the saturation inside the layer, reducing chattering in real implementations [10]

$$S(k) = Ce(k) + \Delta e(k) + K\sum_{j=0}^{k} e(j) \quad (9)$$

Where:

$$\Delta e(k) = e(k-1) - e(k) \quad (10)$$

$e(k)$ is the error signal at the current discrete time step k.

$\sum_{j=0}^{k} e(j)$ is an integral error in discrete-time form, it accumulates past errors to improve steady-state accuracy like the integral action in a PID controller. So, the sliding surface here has proportional, derivative, and integral components of the error. $e(j)$ is the tracking error at the discrete time step (j) C is the coefficient that defines the dynamics of the sliding surface. K is the gain that ensures robustness.

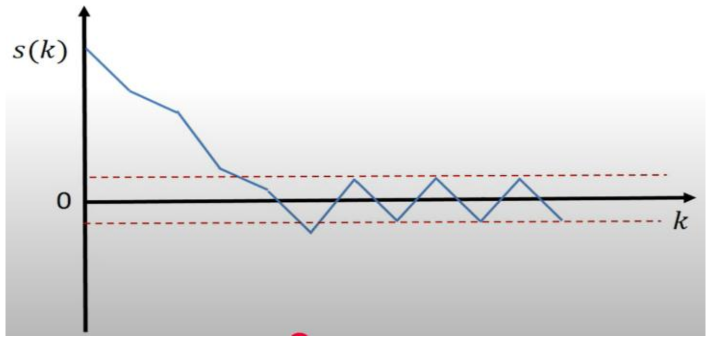

Figure 2: Sliding Surface Graph

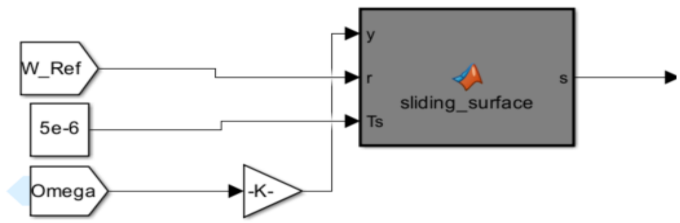

Figure 3: Matlab Function Block for Sliding Surface

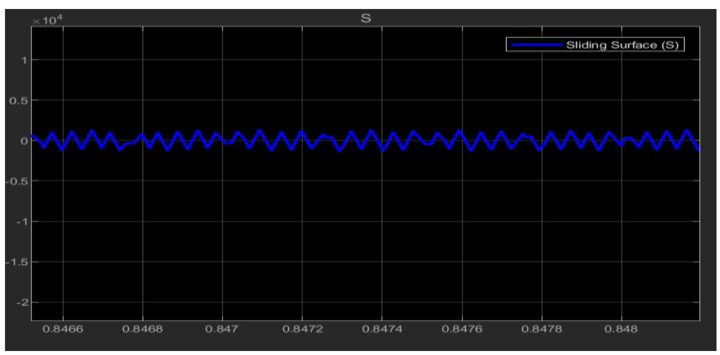

Figure 4: Sliding Surface output

Lyapunov Function Design

We define the standard quadratic Lyapunov function in discrete time as:

$$V(k) = \frac{1}{2}\, S^2(k) \tag{11}$$

Where $S(k)$ is our sliding variable mentioned above. Clearly, $V(k) \geq 0$ for all k, and $V(k) = 0$ only when $S(k) = 0$. The condition for stability in discrete systems requires that the Lyapunov difference satisfies the following:

$$\Delta V(k) = V(k+1) - V(k) < 0, \forall s(k) \neq 0 \tag{12}$$

Substituting the reaching law (7) in (12) we obtain (14).

$$V(k+1) = \frac{1}{2}\left[(1-\eta)s(k) - \eta sat(\frac{s(k)}{\phi})\right]^2 \qquad (13)$$

Thus (13) becomes:

$$\Delta V(k) = \frac{1}{2}\left[(1-\eta)s(k) - \eta sat(\frac{s(k)}{\phi})\right]^2 - \frac{1}{2}S^2(k) \quad (14)$$

When $|s(k)| > \phi$ the saturation reduces to $sat\left(\frac{s}{\phi}\right) = sign(s)$ then we get:

$$s(k+1) = (1-\eta)s(k) - \eta sign\big(s(k)\big) \qquad (15)$$

It follows that:

$|s(k+1)| < |s(k)|$, which implies $V(k+1) < V(k)$.

When $|s(k)| \leq \phi$ the saturation becomes linear, $sat\left(\frac{s}{\phi}\right) = \frac{s}{\phi}$ then we get:

$$s(k+1) = (1-\eta)s(k) - \eta\frac{s(k)}{\phi} \qquad (16)$$

For properly chosen η and ϕ, the coefficient ensures: $|s(k+1)| \leq |s(k)|$, which implies non-increasing $V(k)$. Thus, $S(k)$ remains bounded inside the quasi-sliding band. Figures 5, 6 and 7 depict the simulation results of the Lyapunov function and its discrete derivative, respectively. The monotonic decrease of $V(k)$ and the strictly negative values of $\Delta V(k)$ outside the boundary layer confirm the Lyapunov stability of the PMSM under the proposed DTSMC scheme. The system trajectories converge into the quasi-sliding band, thereby guaranteeing robustness against disturbances and parameter uncertainties

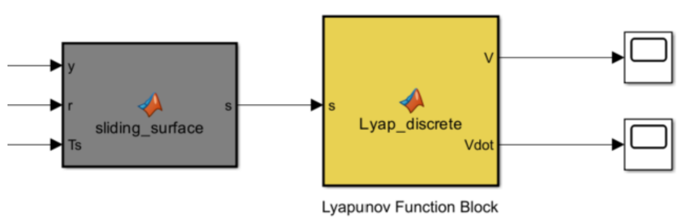

Figure 5: Matlab Function Block for Lyapunov Function design.

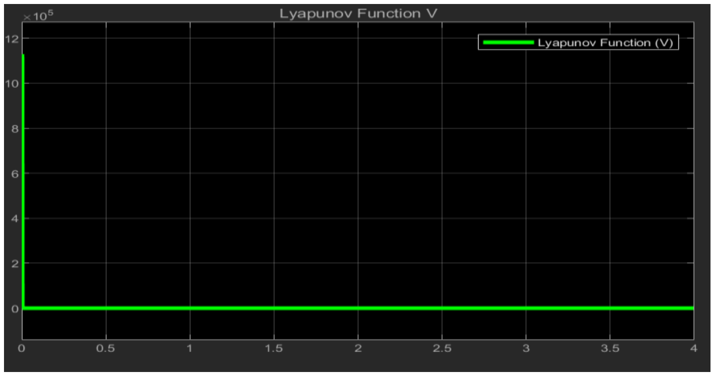

Figure 6: Lyapunov Function $V(k)$

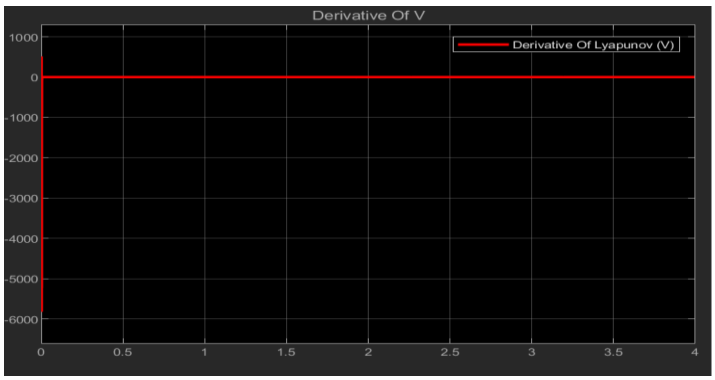

Figure 7: Lyapunov Function $\Delta V(k)$

Design of Control Laws Using Discrete Time Sliding Mode

In designing the control laws, two key aspects were considered: the control input $u(k)$ and the speed tracking error $e(k)$. The control input is formulated to drive the sliding surface $s(k)$ toward zero, while the error indicates how far the system is from the desired trajectory. The fundamental operation of the algorithm is as follows: the error $e(k)$ is first processed by the controller, which then computes the sliding surface $s(k)$ from this error. Finally, the control law uses $s(k)$ to generate the appropriate corrective action.

$$e(k) = \omega_{ref}(k) - \omega_m(k) \tag{17}$$

Where:

- $\omega_{ref}(k)$ is the reference (desired) rotor speed at the discrete instant k.
- $\omega_m(k)$ is the actual measured rotor speed of the PMSM at the same instant.

This error signal $e(k)$ represents the difference between the commanded and actual speed and is the primary feedback quantity used by the discrete sliding mode controller. The control objective is to regulate $e(k) \to 0$, ensuring that the PMSM rotor follows the reference trajectory with high accuracy. From the sliding condition, the control input $u(k)$ must enforce the reaching law (Figure 8). This yields our discrete time sliding mode control law:

$$u(k) = u_{eq}(k) - \left[(1-\eta)S(k) - S(k+1) + \eta sat\left(\frac{S(k)}{\phi}\right)\right] \quad (18)$$

Where: The equivalent control $u_{eq}(k)$ ensures nominal error cancellation.

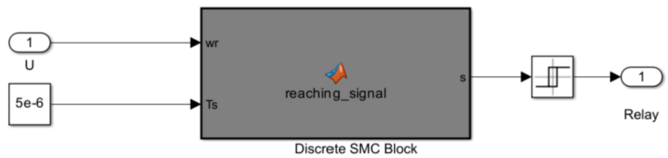

Figure 8: Control Input for reaching law

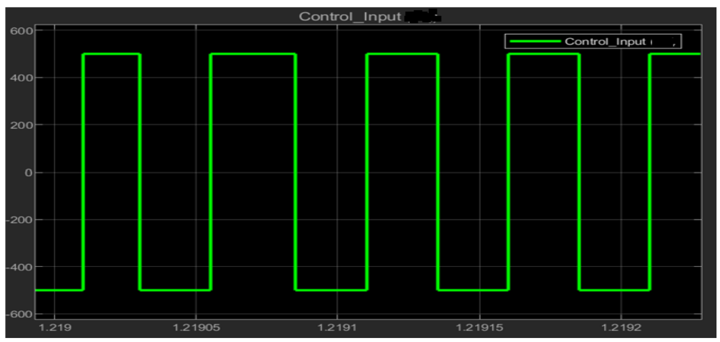

Figure 9: Time-domain response of the control input

Figure 9 illustrates the time-domain response of the control input generated by the proposed discrete-time sliding mode control (DSMC) law. As expected, the control signal exhibits a **high frequency switching behavior,** alternating between positive and negative values of approximately ±500 units. This switching characteristic is inherent to sliding mode control strategies, where the discontinuous control law enforces the sliding condition and drives the sliding surface toward zero. The sharp transitions in the control input indicate that the controller is aggressively compensating for tracking errors, rapidly forcing the system states toward the reference trajectory. This behavior validates the theoretical property of sliding mode control, namely its robustness against disturbances and parameter uncertainties. The magnitude of the switching levels reflects the controller's effort to maintain the sliding surface dynamics specified by the reaching law. Figures 10 and 11 present the speed tracking error responses of the PMSM drive under the proposed discrete time sliding mode control (DTSMC) and a conventional PID controller, respectively. In the case of **DTSMC** (Figure 10), the tracking error rapidly converges to zero and remains effectively negligible throughout the entire simulation horizon. This confirms that the sliding surface design and reaching law enforcement successfully eliminate steady-state error and guarantee robust tracking performance. The flat error trajectory demonstrates the insensitivity of the DTSMC to model uncertainties and external disturbances, which is a defining property of sliding mode control. In contrast, the **PID controller** (Figure 11) exhibits a relatively large initial tracking error, followed by a gradual decay toward zero. Although the PID control reduces the error over time, it fails to eliminate it completely, and small steady-state deviations remain. This slower convergence highlights the limitations of PID in handling nonlinear dynamics and disturbances compared to DTSMC.

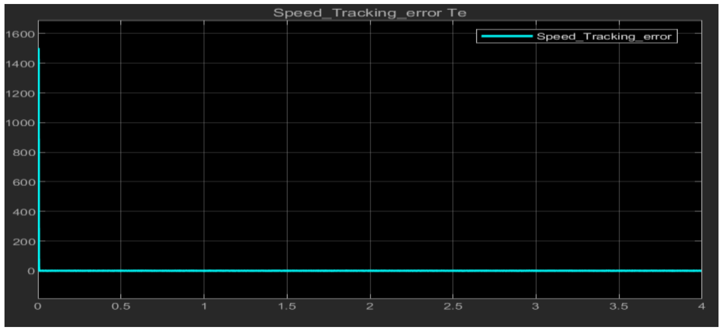

Figure 10: Speed tracking error response (DTSMC)

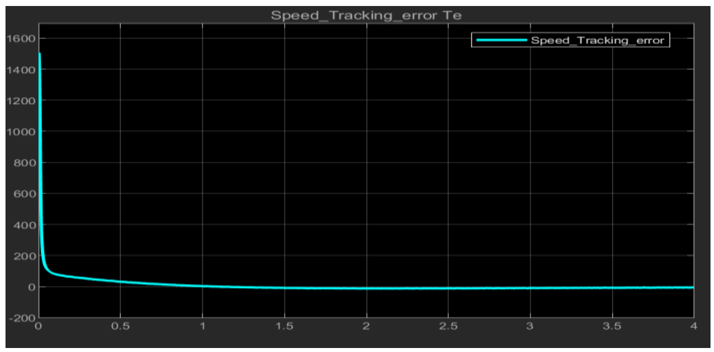

Figure 11: Speed tracking error response (PID)

SIMULATION RESULTS AND DISCUSSIONS

DTSMC Based Control Performance Vs. PID Based Control Under Different Load Types

The simulation results clearly highlight the advantages of the proposed Lyapunov–Discrete Time Sliding Mode Control (DTSMC) strategy over conventional PID-based Field Oriented Control (FOC). Across multiple operating conditions, including constant speed under constant torque (Figures 12 and 13), step changes in reference speed (Figures 14 and 15), ramp trajectories (Figures 16 and 17), oscillatory torque (Figures 18-23), parameter mismatches (24 and 25), and torque load disturbances (Figures 26 and 27)—the DTSMC consistently demonstrated superior tracking accuracy and robustness. Compared to PID control, the DTSMC maintained smaller tracking errors and faster recovery following load disturbances. For example, under step changes in load torque, the PID-controlled system exhibited significant overshoot and longer settling times, while the DTSMC rapidly rejected disturbances and stabilized the speed trajectory. Similarly, under parameter mismatches in Figure 27 (e.g., variations in stator resistance and flux linkage), the DTSMC preserved performance with minimal degradation, whereas the PID controller showed notable tracking errors and instability (Figure 26). These results validate the robustness of the sliding surface design and the Lyapunov-based stability proof. One key implication is that the DTSMC formulation, by leveraging the difference term in its reaching law, is well-suited for digital implementation and avoids excessive sensitivity to discretization. However, the results also reveal a trade-off: while the discontinuous switching element ensures robustness, it introduces chattering in the control effort, particularly visible in the Line voltage command signals (Figures 12-27). Although not destabilizing in this study, chattering may excite inverter harmonics or mechanical resonances in practical systems. Boundary layer methods or higher-order SMC techniques could be considered in future work to mitigate these effects. Overall, combining Lyapunov stability analysis with a discrete SMC reaching law provides both theoretical guarantees and practical robustness, addressing the limitations of conventional controllers that lack resilience to parameter variations and external disturbances.

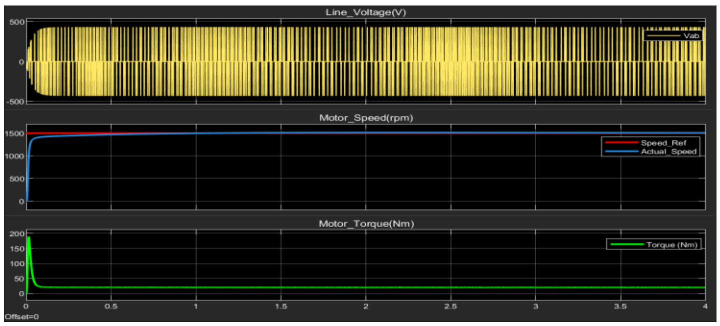

Figure 12: Constant Speed/Constant Torque (PID)

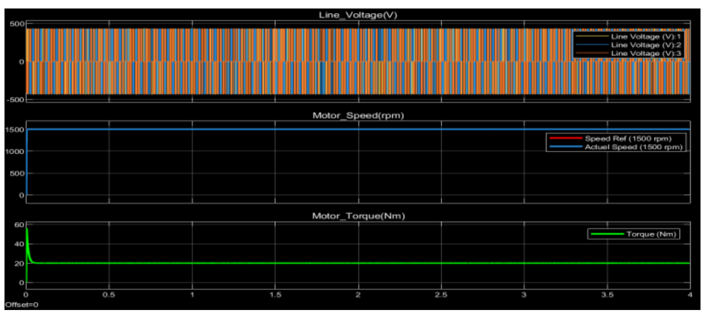

Figure 13: Constant Speed/Constant Torque (DTSMC)

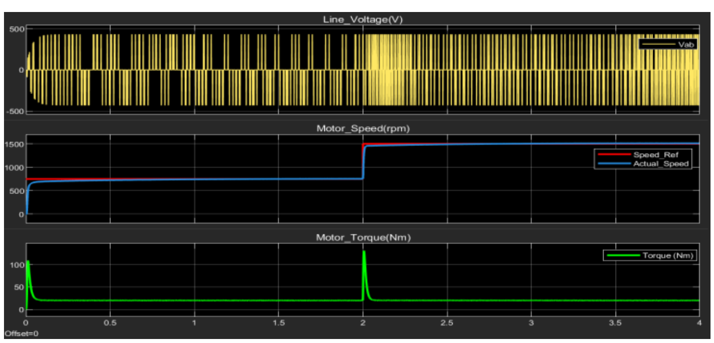

Figure 14: Step Speed/Constant Torque (PID)

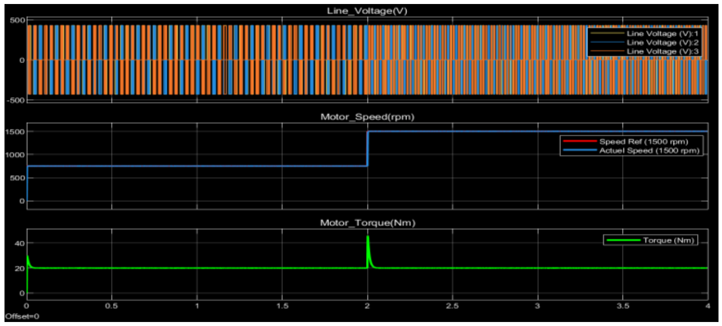

Figure 15: Step Speed/Constant Torque (DTSMC)

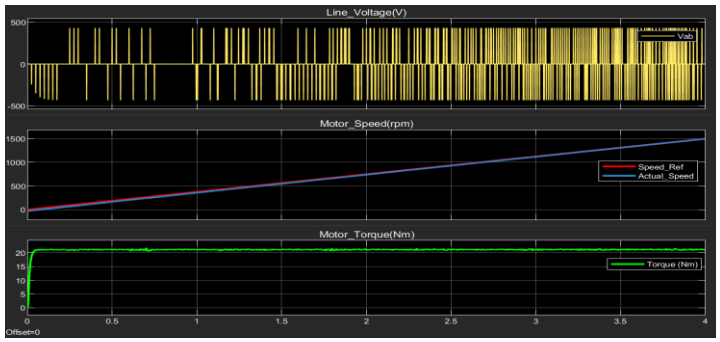

Figure 16: Ramp Speed/Constant Torque (PID)

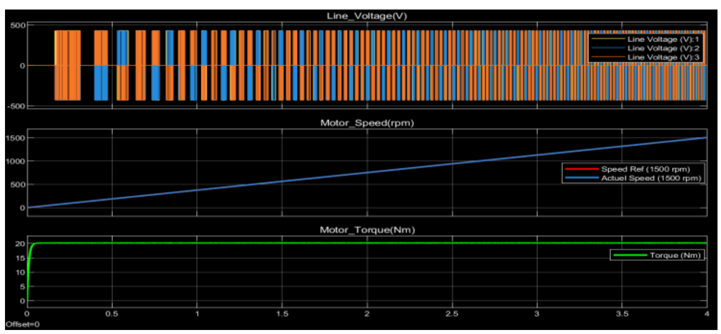

Figure 17: Ramp Speed/Constant Torque (DTSMC)

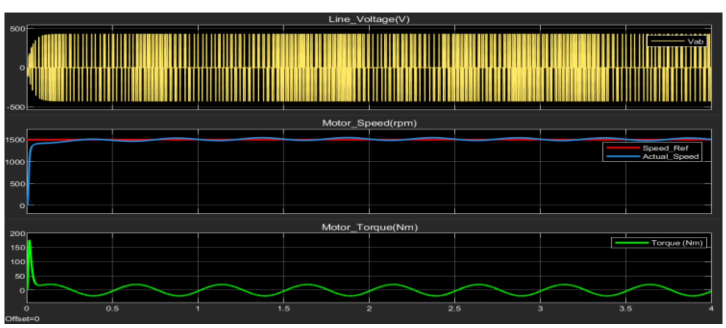

Figure 18: Constant Speed/Oscillatory Torque (PID)

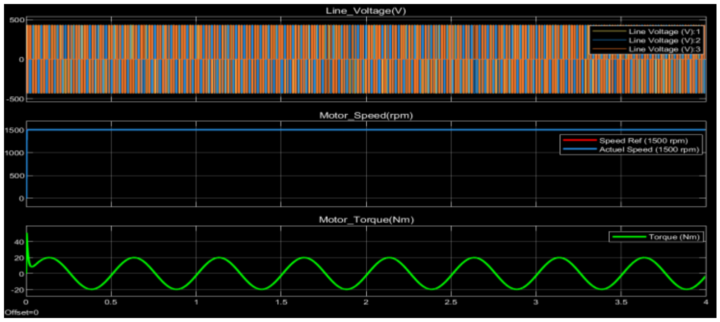

Figure 19: Constant Speed/Oscillatory Torque (DTSMC)

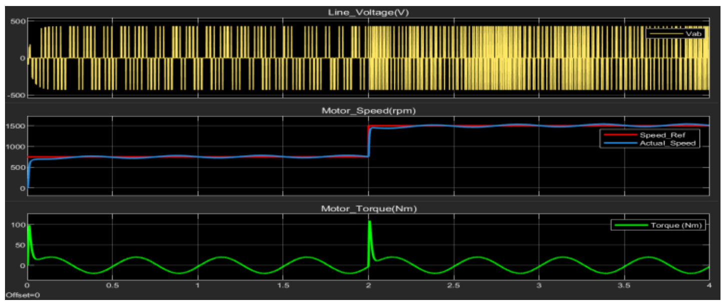

Figure 20: Step Speed/Oscillatory Torque (PID)

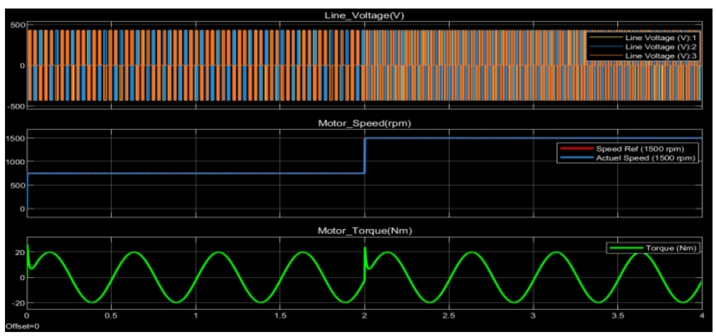

Figure 21: Step Speed/Oscillatory Torque (DTSMC)

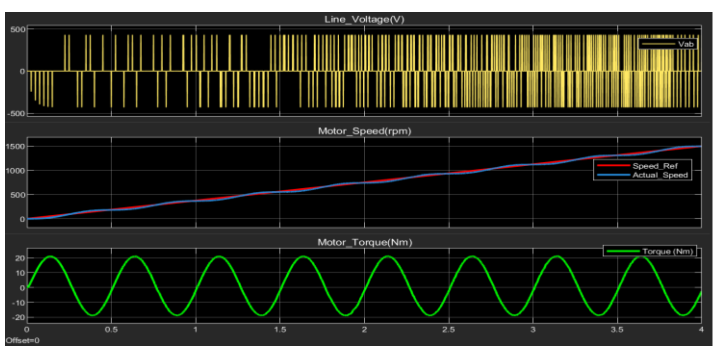

Figure 22: Ramp Speed/ Oscillatory Torque (PID)

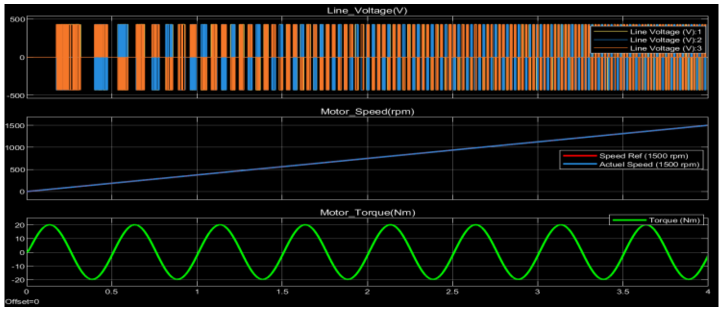

Figure 23: Ramp Speed/ Oscillatory Torque (DTSMC)

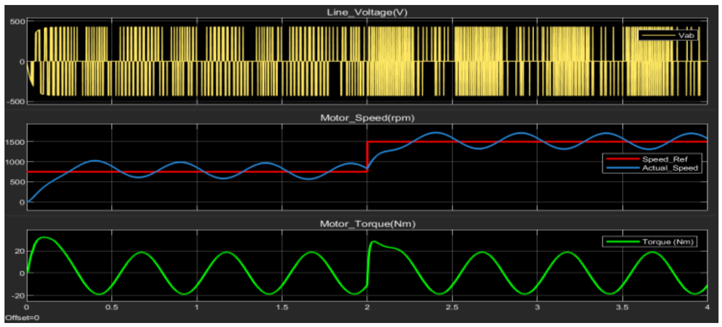

Figure 24: Parameters mismatched (PID)

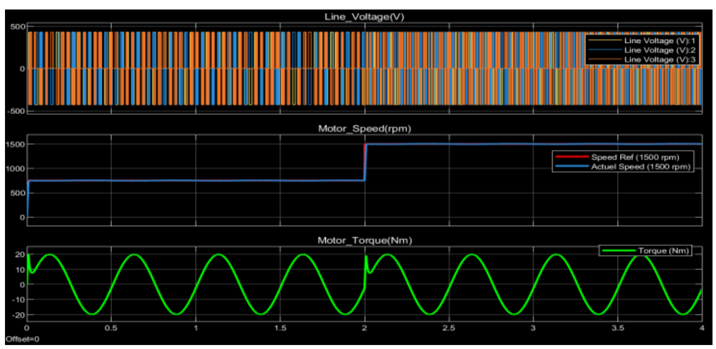

Figure 25: Parameters mismatched (DTSMC)

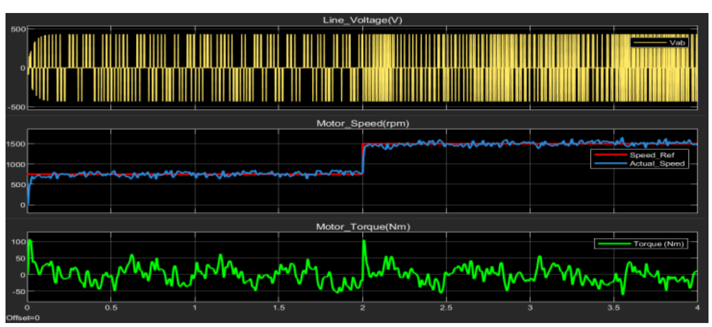

Figure 26: Torque Load Disturbances (PID)

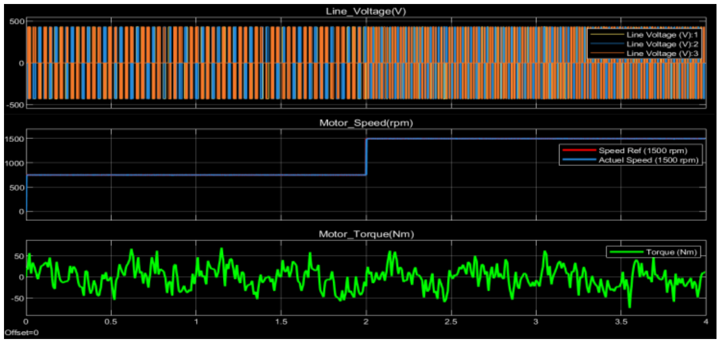

Figure 27: Torque Load Disturbances (DTSMC)

CONCLUSION

This paper proposed a hybrid control strategy that integrates Lyapunov-based control with Discrete Time Sliding Mode Control (DTSMC) for PMSM speed regulation. By designing quadratic Lyapunov functions for stability and discrete sliding surfaces for robustness, the approach ensures convergence of tracking errors, robustness to load disturbances, and resilience against parameter mismatches. Simulation results under a variety of operating conditions demonstrated that the proposed DTSMC significantly outperforms PID-based FOC in terms of tracking accuracy, disturbance rejection, and robustness to uncertainties, while remaining feasible for digital real-time control. The main contributions of this work are: (1) the formulation of a discrete SMC reaching law tailored for digital implementation, (2) the integration of Lyapunov stability guarantees robust disturbance rejection, and (3) comprehensive evaluation of robustness under load torque variations and parameter mismatches. Future research may extend this approach to other motor types such as Induction Machines and multi-motor drive systems, explore higher-order SMC algorithms to reduce chattering, and investigate experimental implementation on embedded motor control hardware with real-time switching constraints.

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