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The Sun and Moon are Larger by About 1.4% than we Think

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ABSTRACT

This is a simple geometric modification to the currently recorded sizes of the sun and moon. The concept is based on the well-known observation that when looking at an object, it appears to progressively taper off toward its distant end. Spherical objects are no exception, even though their apparent tapering is not readily noticeable due to their unique geometry. As shown in this study, when looking at the solar disc or at the moon, we are actually looking at the base side of a seemingly slightly egg-shaped sun or moon. Given that the sun and moon are spherical in reality, the geometrically adjusted sizes show that each is about 1.4% bigger in volume than currently thought.

Keywords: Sun, Moon, Angular Size, Solar Disc, Face of the Moon, Apparent Tapering, Small Angle, 3D Objects, Oval, Egg-shaped, Depth Perception, Sphere, Relative Size.

INTRODUCTION

Angular size is a measure normally used to show how large a celestial body appears from earth. The angular size, also known as the apparent size, angular diameter, or apparent diameter, is the amount of space the object occupies in the observer's field of view. The angular diameters of celestial bodies as seen from earth are typically very small. Therefore, the Small Angle Formula can be used to calculate the actual size of the object by measuring its angular size and finding the distance to it [1, 2].

Let us now limit our discussion to the sun and moon. The sun and moon, which appear to be the biggest heavenly bodies in the earth's sky, have approximately equal apparent sizes of around 0.5° [3]. However, at present, the angular sizes of the sun and moon and their respective distances are measured using complex and precise instruments. Using the Small Angle Formula, their actual sizes are hence obtained. Now the question is whether the current apparent and actual diameters of the sun and moon represent their respective spherical sizes. We must consider this question in light of the well-known observation that objects appear to taper off progressively toward the distant end when viewed. But due to the unique geometry of the sphere, its apparent tapering is not readily noticeable. Nevertheless, the apparent tapering of the sun and moon is worked out in this study, and their sizes are consequently adjusted.

Why Apparent Tapering of the Sphere is not Readily Noticeable

Our eyes gather information about the size, shape, location, brightness, clarity, and movement of objects around us, which are then displayed as two-dimensional images on the retina. Our brain perceives the visuals as three-dimensional with the help of depth perception. The relative size is one of the indicators for depth perception. In other words, objects of the same size are perceived as being closer when they are larger and distant when they are smaller [4]. For example, Figure 1(a & b) [5, 6] illustrates how road tunnels, which are typically of regular width and height, appear to taper off gradually in the forward direction. However, due to two unique aspects of the sphere's geometry, apparent tapering is not easily obvious for spherical objects. The first aspect is that for the sphere, unlike for regular 3D objects, the relative size indicator of depth perception is reversed, as illustrated in Figure 2. The nearest to the viewer will be a point of zero angular size. Then geometrically, the sphere enlarges in the forward direction at a rate much higher than that of apparent tapering. The second aspect is that the sphere as a unique 3D object does not have any edges or vertices. It is a round 3D shape with all the points on its surface at equal distances from the center [7]. How much of the sphere can be seen depends on the angular diameter [8]. For very small angular diameters, such as that of the sun or moon, we can see almost a whole hemisphere. So, while the sphere is rotating, as illustrated in Figure 2, an apparently tapered hemisphere will be continuously generated symmetrically around the line connecting the view point and the center of the sphere. Therefore, the view will constantly be the same circular disc, which makes the apparent tapering unnoticeable.

a b

Figure 1: (a) Sharqiyah Tunnel2, Oman [5], (b) Wadi Al Helo Tunnel, UAE [6].

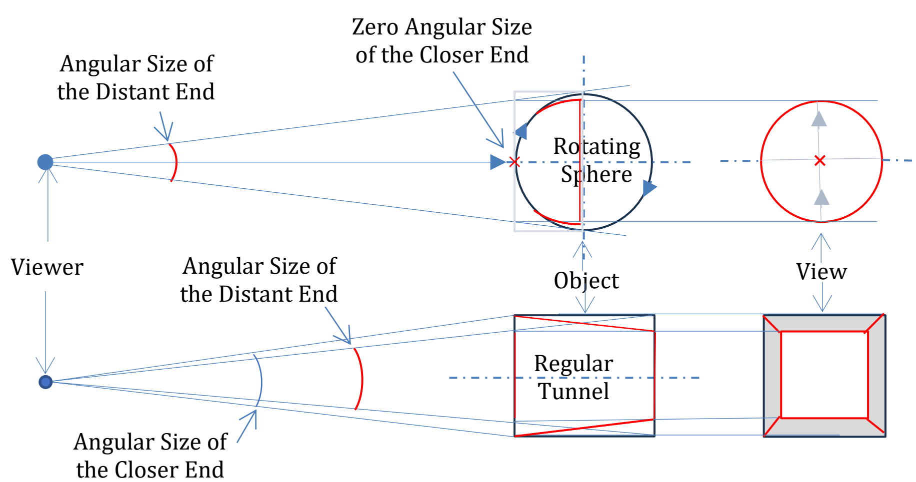

Figure 2: Apparent tapering of a sphere compared to a regular tunnel.

Apparent Tapering of the Sun and Moon and their Adjusted Sizes

First, to form a rough idea about how the sun and moon taper off, let us imagine that a rotating sphere is circumscribed by a right circular hollow cylinder, as illustrated in Figure 3. Now, looking down the axis of the cylinder, the angular diameter is assumed to be as small as that of the sun or moon. In this case, the observer can see almost up to the middle plane [8], where the cylinder's internal wall is tangential to the sphere. The cylinder will appear to taper off gradually toward the distant end in relation to the closer end. Thus, we can imagine that the rotating sphere contained within the cylinder will likewise eventually taper and apparently become egg-shaped, as illustrated two-dimensionally by the oval shape in Figure 3. The base side of the apparently egg-shaped sphere will be seen as a circular disc, as represented by the yellow circle whose apparent size is equal to the minor diameter of the tapered sphere. The white ring around it is a view of the apparently tapered front half of the cylinder up to the minor axis of the tapered sphere. All of the back half, including the middle plane, will be blocked by the base of the apparently egg-shaped sphere.

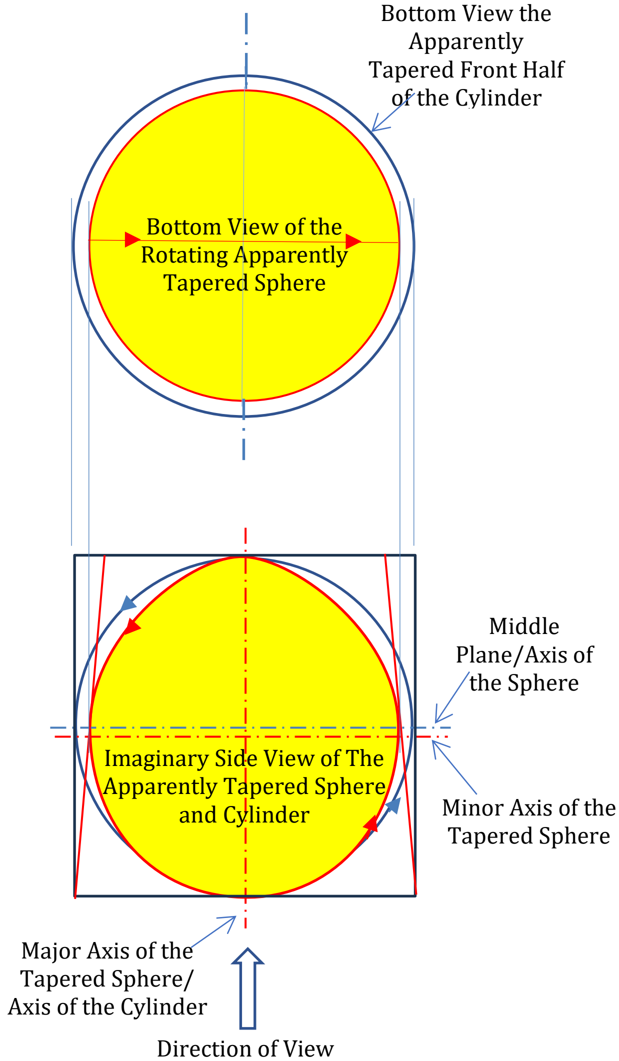

Figure 3: Apparent tapering of a sphere circumscribed by a right circular hollow cylinder.

Let's now examine the cylinder's tapering, followed by the combined cylinder and circumscribed sphere's tapering. In Figure 4, the side view of the cylinder's front half is geometrically represented by the rectangle ABCD. The radius and depth of the cylinder are equal to the radius of the circumscribed sphere, R_1. The distant end of the front half of the cylinder is the middle plane, which is blocked, as explained above, by the plane of the minor axis of the apparently tapered sphere. However, for the very small apparent sizes of the sun or moon, the difference between the apparent size of the minor axis and that of the middle plane is not expected to be that significant. Therefore, the angular size of the distant end of the half cylinder, θ_2, can be approximated to that of the minor axis of the apparently tapered sun or moon. The rays of the angle θ_2 intersect with the line AB at points E and G, as seen in Figure 4. Thus, the relative size comparison of the cylinder's far and closer ends equals the linear size of segment EG to that of line AB. The front view, represented by the two concentric circles on the right, can then be drawn by projecting the points A, E, F, G, and B horizontally. The two concentric circles represent how the image lands on the retina. The grey annulus is the seemingly tapering inner wall, and our brain will interpret the outer circle, with radius R_1, as the closer end and the inner one, with radius R_2, as the farther one.

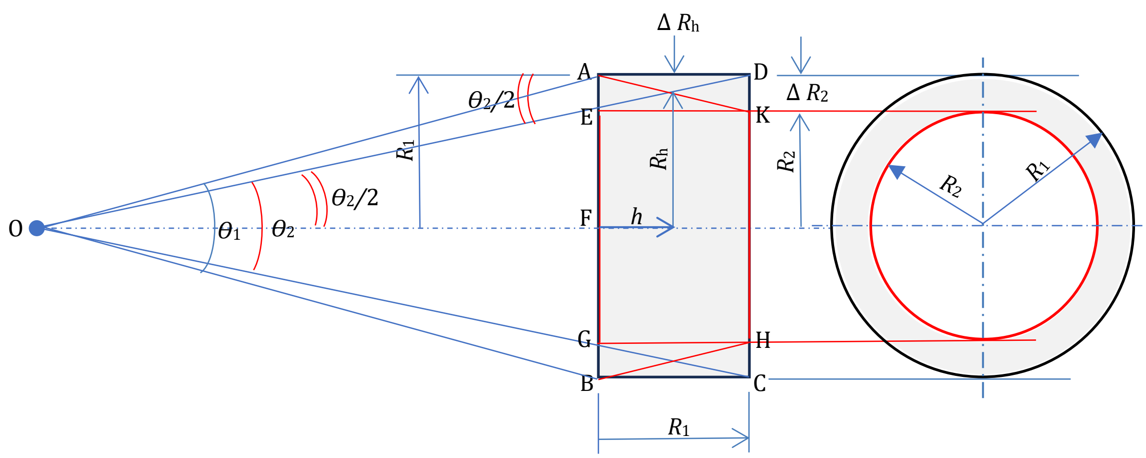

Figure 4: Apparent tapering of a right circular hollow cylinder.

It is obvious that apparent tapering of straight regular objects is of a constant gradient. In the triangle ADE, as shown in Figure 4, AE is equivalent to ΔR_2. which is the difference between R_1 and R_2. AD is equivalent to the depth R_1. Angle ADE equals $\theta_2/2$, since angle DOF, which is half of θ_2, alternates with angle ADE. The tapering gradient, m, of the aforementioned half cylinder can, hence, be found using the following formulae:

$$\Delta R_2 = R_1 - R_2 \tag{1}$$

$$m = \Delta R_2 / R_1 = AE / AD = tan(\theta_2/2) \tag{2}$$

As assumed above, Θ_2 is equal to the apparent size of the sun or moon, which is a small angle. Hence, provided that Θ_2 is in radians, using the Small Angle Formula [9], Equation (2) can be rewritten as follows:

$$m = tan(\theta_2 /2) \approx (\theta_2/2) \tag{3}$$

$$\therefore\ m \approx (\theta_2/2) \tag{4}$$

The amount of apparent tapering, ΔR_h, at any depth, h, can hence be expressed as follows:

$$\Delta R_h = mh \approx (\theta_2/2)h \tag{5}$$

Apparent tapering rate, i.e. apparent tapering per unit length of the radius R_1 at any depth h, is hence:

$$\Delta R_h/R_1 = (\theta_2/2)h/R_1 \tag{6}$$

Let us now consider the apparent tapering of the combined cylinder and circumscribed sphere. The circle of radius R_1 in Figure 5 (a) is a two-dimensional geometric representation of the sphere. The oval shape represents the seemingly tapering sphere. The rectangle ABCD, which appears to taper to the trapezoid ABEF, represents the front half of the cylinder. Starting at point O, the angle φ is obtained by rotating the radius R_1 in a counterclockwise direction for one revolution. The circumscribed sphere represented by the circle in Figure 5 (a) can alternatively be thought of as a set of successively connected spherical circles that are aligned along the axis of the cylinder. In any perpendicular plane along the axis of the cylinder, e.g., plane XX at the random depth h, the radius of the spherical circle is geometrically equal to $R_1 \sin \varphi$. This spherical circle is contained within the cross section of the right circular cylinder in the same plane as illustrated in Figure 5 (b). Obviously, the tapering per unit length of the cylinder's radius $\Delta R_h/R_1$, as given above by Equation (6), will apply to the spherical circle contained in the same plane

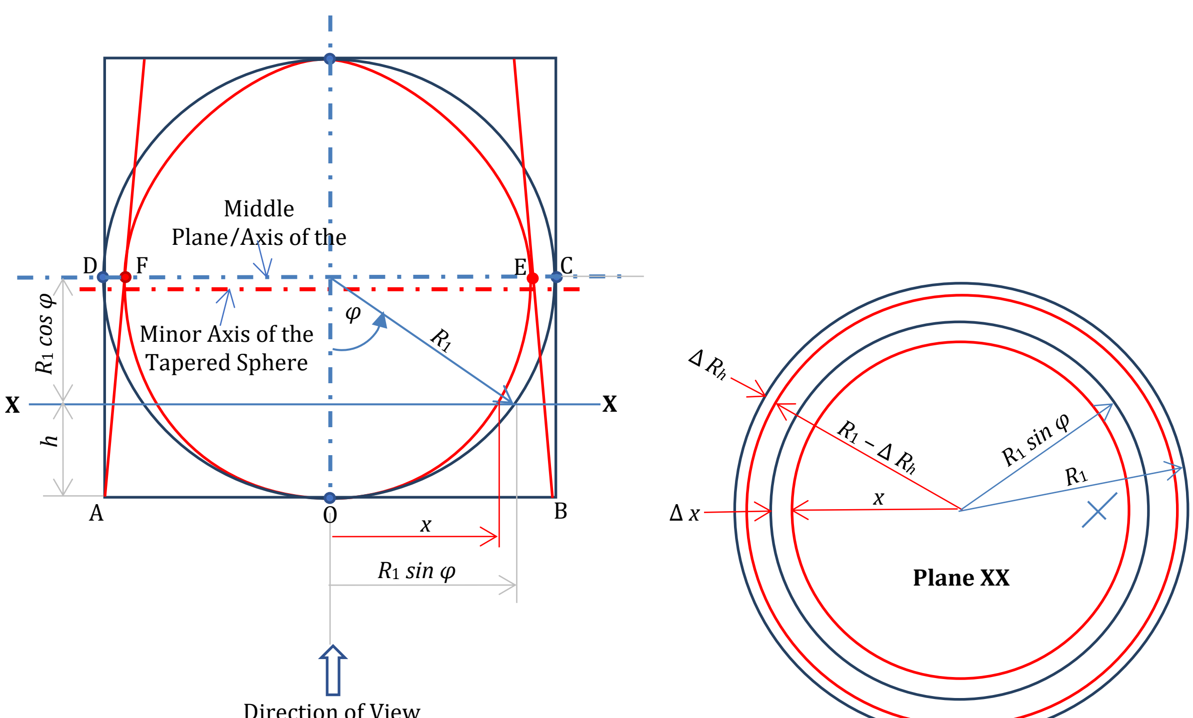

Figure 5: (a) The combined cylinder and circumscribed sphere's tapering. (b) Plane XX.

With reference to Figure 5 (a & b), it is possible to express the variables h, $\Delta R_h/R_1$, Δx, and consequently x in terms of the independent variable angle φ, the geometrically adjusted radius of the sphere R_1, and its apparent size Θ_2.

As illustrated in Figure 5 (a), the depth, h, is given by:

$$h = R_1 - R_1 \cos\varphi = R_1(1 - \cos\varphi) \tag{7}$$

From Equations (6) and (7), the apparent tapering of the cylinder per unit length of the geometric radius R_1 is:

$$\Delta R_h/R_1 = (\theta_2/2)R_1(1 - \cos\varphi)/R_1 = (\theta_2/2)(1 - \cos\varphi) \tag{8}$$

In any given plane, the difference, Δx, between the geometric radius of the spherical circle and its tapered radius can be obtained by multiplying the tapering per unit length given in Equation (8) by the spherical circle's radius, $R_1 \sin\varphi$, as follows:

$$\Delta x = (\theta_2/2)(1 - \cos\varphi)R_1 \sin\varphi \tag{9}$$

From Figure 5 (b):

$$x = R_1 \sin\varphi - \Delta x \tag{10}$$

From (9) and (10):

$$x = R_1 \sin\varphi - (\theta_2/2)(1 - \cos\varphi)R_1 \sin\varphi = R_1 \sin\varphi\ [1 - (\theta_2/2) + (\theta_2/2)\cos\varphi] \tag{11}$$

From (11):

$$x = (1 - \theta_2/2)R_1 \sin\varphi + (\theta_2/2)R_1 \sin\varphi\ \cos\varphi \tag{12}$$

Equation (12) shows that the spherical circle's radius x is a function of the variable angle φ, the geometric radius of the sphere, R_1, and the apparent size of the sun or moon, Θ_2. As seen in Figure 5 (a), the spherical circle's radius x increases from zero at φ equal to zero to a maximum equal to the minor radius of the oval shape, R_o, at a value of φ close to 90°. The minor radius R_o represents the currently recorded radius of the sun or moon. The plane of the minor radius R_o is a little bit below the center of the sphere. This plane, as explained earlier and as illustrated in Figures 3 and 5 (a), blocks all of the top part, including the central geometric plane of the combined sphere and cylinder. The angle φ at which x is maximum, i.e., at which x is equal to R_o, can be found by equating the first-order derivative of Equation (12) to zero [10].

Using differentiation of trigonometric functions [11], the derivative of Equation (12) is:

$$dx/d\varphi = (1 - \theta_2/2)R_1 \cos\varphi + (\theta_2/2)R_1(\cos^2\varphi - \sin^2\varphi) \tag{13}$$

From basic trigonometric identities, $sin^2\varphi$ + $cos^2\varphi$ = 1 for any angle φ. Therefore, $sin^2\varphi$ in Equation (13) can be replaced with ($cos^2\varphi$ – 1). Equating dx /dφ to zero, we then obtain:

$$(1-\theta_2/2)R_1 \cos\varphi + (\theta_2/2)R_1(2cos^2\varphi - 1) = 0 \quad (14)$$

Equation (12) can be rewritten as:

$$\theta_2 cos^2\varphi + (1-\theta_2/2)\cos\varphi - \theta_2/2 = 0 \quad (15)$$

The value of the unknown x in a quadratic equation of the form $ax^2 + bx + c = 0$ may be found using the Quadratic Formula [12], $x = (-b \pm \sqrt{}(b^2 - 4ac))/(2a)$, where the coefficients a, b, and c are real or complex numbers. Equation (15) above is a quadratic equation with the coefficients $a = \Theta_2$, $b = (1 - \Theta_2/2)$, and $c = (-\Theta_2/2)$, where Θ_2 is in radians and $cos\ \varphi$ is the unknown x. Therefore, $cos\ \varphi$ can be obtained as follows:

$$\cos\varphi = \{-(1-\theta_2/2) \pm \sqrt{(1-\theta_2/2)^2 - 4\theta_2(-\theta_2/2)}\}/2\theta_2 \quad (16)$$

Equation (16) can be rewritten as below:

$$\cos\varphi = \{-(1-\theta_2/2) \pm \sqrt{(1-\theta_2/2)^2 + 2(\theta_2)^2}\}/2\theta_2 \quad (17)$$

Hence,

$$\varphi = \cos^{-1}\langle\{-(1-\theta_2/2) \pm \sqrt{(1-\theta_2/2)^2 + 2(\theta_2)^2}\}/2\theta_2\rangle \quad (18)$$

RESULTS AND DISCUSSION

In the above equation, Θ_2, as was previously supposed, is alternately the apparent diameter of the sun and moon. From the Sun Fact Sheet [13], the apparent diameter of the sun, Θ_2, is 1919 arc seconds. For the moon, the apparent diameter is 1896 arc seconds [14]. Converting the arc seconds into radians [15], the values of Θ_2 for the sun and moon are 0.009304 and 0.009192 radians, respectively. In the domain $0 \le \varphi \le 90°$, the angle φ at which x is maximum, i.e., $x = R_o$, can hence be obtained for the sun and moon by substituting their above respective apparent diameters in Equation (18). Equation (18) as a cosine function has a range between -1 and 1 [16]. Considering the values within the range, the values of the angle φ at which x is maximum ($x = R_o$) for the sun and moon are 89.735 and 89.732°, respectively.

Now, substituting the above respective values of the angle Θ_2 in radians and the angle φ in degrees in Equation (12), we obtain a ratio of the currently recorded radius, R_o, to the geometrically corrected radius, R_1, of the sun and moon, respectively, as follows:

For the sun:

$$R_0/R_1 = 0.99536 \quad (19)$$

From (19):

$$R_1/R_o = 1.00466 \quad (20)$$

From (20), the ratio of the geometrically adjusted volume V_1 to the currently recoded volume V_0 of the sun is Hence:

$$V_1/V_o = (1.00466)\ \text{^}3 = 1.014 \quad (21)$$

Hence:

$$(V_1 - V_o)\ /V_o\% = (1.014 - 1) * 100/1 = 1.41\% \quad (22)$$

Now, as evident from Equation (22), the sun's geometrically adjusted volume is larger than the currently recorded volume by 1.4%. Therefore, the sun's mass, based on its mean density, is also 1.4% larger. With reference to the Sun Fact Sheet [14], the sun/earth mass ratio is thus increased from 333,000 to 337,662. In other words, the geometrically adjusted mass of the sun is bigger by 4,662 times the mass of Earth.

For the moon, and as the values of the angles Θ_2 and φ are almost equal to those of the sun, the following ratios of the radii and volumes are almost equal to those of the sun:

$$R_0/R_1 = 0.99541 \quad (23)$$

From (23):

$$R_1/R_o = 1.00461 \quad (24)$$

From (24)

$$V_1/V_o = (1.00461)\ \text{^}3 = 1.014 \quad (25)$$

From (25), the ratio of the geometrically adjusted volume V_1 to the currently recoded volume V_0 of the moon is Hence:

$$(V_1 - V_o)\ /V_o\% = (1.014 - 1) * 100/1 = 1.39\% \quad (26)$$

Similar to the sun, the moon's geometrically adjusted volume is approximately 1.4% greater than its currently recorded volume. Therefore, the moon's mass, based on its mean density, is also 1.4% larger. But according to the Moon Fact Sheet [15], the moon/earth ratio is just 0.0123. As a result, the adjusted mass is bigger by only 0.000172 times that of Earth.

CONCLUSION

The fundamental idea of this work was the apparent tapering of objects toward their far end. As explained in Section 2 of this paper, the apparent tapering of spherical objects is difficult to see, but it can be realized when a sphere is geometrically encased in a right circular cylinder or even a tunnel. Following the apparent tapering of the cylinder or tunnel toward its far end, one can imagine the eventual tapering of the encased sphere. This idea was used in Sections 3 and 4 to calculate the apparent tapering of the sphere encircled by a right circular cylinder. It was easy to calculate the tapering rate of any plane of the cylinder along its axis because the tapering of regular, straight, hollow objects is obviously a linear function of the plane's depth with respect to the closer end. Thus, the tapering amount is the product of the depth and the tapering rate. It goes without saying that the radius of the cylinder will taper uniformly over the plane's radius. The sphere's confined portion will thereafter taper at the same pace. Then, with the aid of basic geometry, it has been demonstrated that the volume of the sun and moon is 1.4% greater than what is commonly understood. Due to their nearly equal apparent diameters, the sun and moon have shown almost equal corrected to recorded volume ratios.

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