



Rationalizing the Mathematics Requirements for a Master of Biostatistics Program: A Case Study and Commentary

Jesse Troy

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

Steven Grambow

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

Megan Neely

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

Gina-Maria Pomann

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

Clemontina Davenport

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

Marissa Ashner

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

Greg Samsa

Department of Biostatistics and Bioinformatics,
Duke University, Durham NC, USA

ABSTRACT

Within a 2-year Master of Biostatistics program, we reconsidered the mathematics requirements for admission, and also the premise that the most distinguishing feature of a top candidate is deep exposure to mathematics. Our assessment took place within a broad curriculum review intended to enhance alignment: aligning programmatic goals with job skills, aligning the use of mathematics within our curriculum with programmatic goals, aligning our admission criteria with our curriculum, and aligning our application materials with these admission criteria. We developed a specific list of mathematical skills required by the curriculum, and are revising the application materials to include self-report on applicant's exposure to and functional mastery of those skills. We illustrate how functional mastery is operationally defined. Our criteria for identifying top candidates was broadened to include those with especial skills in analytics, biology and/or communication (i.e.,

the three conceptual pillars of our program). Deep mathematical training is one of many ways to become a top candidate. Within STEM fields such as biostatistics, and despite the commonly held assumption that admission criteria should emphasize depth of mathematical training, a systematic analysis suggests that this assumption imposes a gratuitous requirement on applicants. Reconsidering this assumption can help remove unnecessary barriers, reduce challenges, and support success for aspiring or emerging biostatisticians from diverse or multifaceted backgrounds. It is one way to contribute to a more inclusive and equitable learning environment in our STEM discipline. We believe similar programs might benefit from performing this type of analysis and reflection.

Keywords: Biostatistics, curriculum review, mathematics requirements, STEM pipeline.

INTRODUCTION

Lorem A basic premise of program development is that of constructive alignment [1] -- for example, curriculum content should be aligned with overall educational goals, and admission criteria should be aligned with curriculum content. Moreover, educational goals should be realistic. For example, the curriculum of a professional master's program should be focused on tasks that graduates will accomplish on the job (currently, and in the future), and gratuitous requirements should be avoided.

Here, we consider a 2-year Master of Biostatistics (MB) program. Originally approved as a terminal professional degree program, it now serves two types of students. Approximately 30% immediately proceed to doctoral study after graduation. The other 70% enter the workforce. A distinctive feature of the MB program is its broad focus, consistent with a mission to train students who will contribute to interdisciplinary team science, encompassing not only Analysical skills but Biological knowledge and Communication skills as well (i.e., the "ABCs of biostatistics") [2-10]. Graduates often take positions with significant responsibility and autonomy, and our intention is to train students for this type of "high-end" application.

The current case study and commentary pertain to the alignment between curriculum content and admission criteria. Because a biostatistics curriculum is mathematically intensive students must be "good at math", and our question is how that construct can be best operationalized. Previously, our measure of this construct was based on grades and course lists. At a minimum, we required calculus through multi-variable calculus and linear algebra (i.e., the two main languages of instruction), with the default expectation that the multi-variable calculus grade was an A. Moreover, top candidates were expected to have additional coursework in real analysis, differential equations, etc. In other words: (1) the primary criterion for basic competency was demonstrated excellence in multi-variable calculus; and (2) a requirement for being considered as a top candidate was deep training in mathematics, and the deeper the better. Other master's programs apply similar admission criteria.

We continue to believe that mastery of multi-variable calculus is an excellent measure of basic competency. However, the seemingly intuitive criteria for identifying top candidates had some unintended consequences. We begin by describing those consequences, and then how our admission criteria are being modified in response. We believe similar programs could benefit from performing this type of analysis and reflection.

UNINTENDED CONSEQUENCES

One consequence of our admission criteria was they were hard to meet for anyone who majored in a discipline other than mathematics or statistics. More specifically, students in other disciplines might have the required minimum training but, so long as "more math is better", would not be considered to be top candidates. Indeed, some undergraduate statistics programs, especially those with a strong focus on data science (i.e., a discipline that is closely related to statistics) have modest requirements for their mathematics courses, and thus even the training of some statistics and data science majors was deemed to be problematic. Not only were some potentially excellent candidates missed, but we recognized that the pool of traditional candidates (i.e., mathematics and statistics majors) was too small to fit the ever-increasing demands for biostatisticians.

A related consequence is that some applicants were penalized because of the lack of depth in the course offerings from their undergraduate institutions. For example, a small liberal arts school might offer a single course in multivariable calculus but nothing in linear algebra, nor could the desirable additional exposure to mathematics beyond our minimum be found within that institution.

Another consequence resulted from the recognition that, across institutions, mathematics courses with the same name cover different information at different levels of depth and rigor. This phenomenon was especially challenging to address for applicants from undergraduate institutions outside the United States. Once students entered our program it turned out that a good grade in an undergraduate calculus course didn't necessarily imply functional mastery of calculus -- that is, the ability to use calculus as a problem-solving tool. For our purposes, such students weren't actually "good at math".

RESPONSE

Our response to these unintended consequences began with a comprehensive and ongoing curriculum review. An initial step was to identify the skill set required by our graduates and to revise the curriculum so that (1) each important skill was addressed; and (2) less important skills were deemphasized, eliminated, or made optional. This step led to big-picture changes such as adding and dropping courses, revising the mapping of courses into tracks, etc. These changes supported better alignment between the overall curriculum and overall educational goals (i.e., made concrete by the desired student skill set).

Next, we reviewed how effectively important skills were taught within specific classes (e.g., active learning approaches were encouraged). This was the point at which we assessed the requisite degree of mathematical coverage. For example, our first inference course was revised to replace some (but certainly not all) calculus with demonstrations via simulation, and to replace formal proofs with other intuition-building exercises [9]. This supported the second element of alignment: namely, better alignment between course goals and course content (especially, mathematical content).

Once this initial process of course revision was completed, we considered how mathematics is used within individual courses. Upon reflection, two differing skill sets are needed: (1) the general ability to use mathematics to formulate and solve statistically-based problems; and (2)

mastery of specific prerequisite knowledge required by our program. The second and more specific skill set was simpler to operationally define.

Regarding prerequisite knowledge, we reviewed the courses that a job-bound student would be likely to take (thus, for example, selecting elective courses with an applied focus) and recorded the specific applications of mathematics. To our surprise, this list (Appendix 1) is not voluminous. It contains deep working knowledge of some basic concepts, such as the nature of functions. It also includes a relatively small subset of the material typically covered in an undergraduate course in calculus which focuses on the trajectory of objects as they move through space and similar concepts from physics. Additionally, it includes (1) facility with differentiation and integration, which are required (among others) for manipulating the probability distributions used in statistics; and (2) ability to manipulate functions -- especially, to maximize and minimize them.

A similar phenomenon was observed for linear algebra -- rather than the entire content of a typical linear algebra course, the information that is most important to our program is limited to (1) the ability to manipulate matrices (e.g., to add them, to invert them); (2) the ability to state statistical models in matrix terms (i.e., as this allows ideas to be presented succinctly and in greater generality); and (3) the ability to project high-dimensional spaces onto lower dimensions (i.e., this being critical to statistical modeling, and requiring facility with content such as eigenvalues and eigenvectors).

Regarding the first skill set, the underlying construct (i.e., general ability to use mathematics to formulate and solve statistically-based problems) was straightforward to state, but not necessarily trivial to assess. Presumably, having good grades in undergraduate courses in mathematics, or which extensively use mathematics, is a necessary but not fully sufficient condition for assessing this construct, because students can master the technical elements of mathematics as an exercise in symbol manipulation but not truly understand the underlying concepts (and, thus, not be able to extend their knowledge to different problems).

In any event, we had created a list of what students should be able to do: namely, use calculus to accomplish a specific set of tasks, use linear algebra to accomplish another specific set of tasks, and apply mathematics more generally to define and solve statistical problems. For each of these constructs, our admission materials could only provide a rough surrogate, albeit a surrogate that we hoped to improve.

MODIFYING THE ADMISSION PROCESS

Considering the above, we have made two major changes to the admission process. The first is an attempt to better translate course titles into an assessment of the mastery of the specific items in Appendix 1. Toward this end, we had previously asked that applicants describe the level of rigor associated with their mathematically-related courses, and many applicants had additionally copied course descriptions from a catalog. Starting with the next application cycle, we will enhance this report by (1) asking students to self-rate their facility with the items in Appendix 1; and (2) provide example responses that illustrate how we conceptualize mastery (see Appendix 2). In other words, we will ask applicants to provide more specific self-reports about their mastery of the content that will be important to their success in our program, and also provide examples which help them calibrate these reports.

The second change was to reconsider what it means to be a top candidate. Rather than simply ranking students on the number of mathematically-related courses taken as an undergraduate (1) we continue to require all successful applicants to demonstrate basic competency in multi-variable calculus and linear algebra; and (2) for applicants satisfying this first condition, we now define a top candidate as someone with especial skills in analytics, biology and/or communication (i.e., the three conceptual pillars of our program). In other words, additional mathematical training remains one way to become a top candidate, but isn't the only one. Indeed, we had previously been making such judgments on an *ad hoc* basis, and have found that doing so in a systematic, transparent and reproducible basis helps to simplify the application review process as well as making it more consistent.

DISCUSSION

We have described the process by which our admission requirements around mathematics were systematically reviewed and updated. Our intention is to more effectively align the use of mathematics in our curriculum with broader programmatic goals, to better align our admission criteria with how mathematics is used, and to better align the information in our application materials with these admission criteria. Reconsideration of admission criteria was a final step in a broader process of curriculum review. We were encouraged to find that the specific list of mathematics requirements was of modest length, and favored depth of knowledge over breadth. We also recognized a mismatch between this list of mathematics requirements and the information currently available on our application (e.g., lists of undergraduate math-related courses and their grades), and are revising our application materials accordingly. In part because of the ability to perform web searches, to use generative artificial intelligence, etc., we do not directly evaluate mathematical skills within the application (e.g., we don't ask applicants to answer questions such as those in Appendix 2), but instead provide information about our interpretation of functional mastery of mathematical skills to help support their own self-evaluation. An additional benefit of this review is that potential applicants will receive more specific information about the preparation they need to succeed in our program -- for example, to assist them in their selection of undergraduate courses. Indeed, we are in the process of updating the descriptions of our program (e.g., on our website) to make our mathematical requirements more explicit.

Although not the primary rationale for revising our admission process, we note that our approach is relevant to the question of the "shrinking pipeline in the STEM disciplines" [11-13]. As applied to mathematics and statistics, this pipeline begins in middle school (and before) with students who are interested in mathematics. Some drop out of the pipeline in high school, because math courses are uninteresting, irrelevant, poorly taught, and/or too hard for their level of preparation. The same applies to their undergraduate experience, especially if they encounter a "weed-out course", causing them to decide against a mathematically-related major. Students with fewer math courses or having non-math majors are less likely to pursue graduate study in a STEM field and ultimately receive a graduate degree. At each step, these challenges disproportionately fall on students from disadvantaged groups, due to a combination of beliefs (e.g., stereotyping), individual behavior, institutional behavior, resources (both formal and informal) and inadequate support. The ultimate result is that too few students ultimately succeed in the STEM fields, and also that the distribution of those who do is skewed toward students from more advantaged backgrounds.

The points at which our program can intervene fall late in the process: (1) by broadening admission requirements; (2) by enforcing consistency between admission requirements and the curriculum; and (3) by striving to provide a supportive environment to all students, recognizing that doing so requires due consideration of individual characteristics and social context. Here, we have focused on the first two elements of the list: in essence, by returning selected non-math majors who are traditionally assumed to have exited the pipeline back into it, and by providing a curriculum for which they are adequately prepared. Broadening admission requirements is especially reasonable for a "team science discipline" such as biostatistics where a successful practitioner could, for example, build upon deep expertise in biology and competence in mathematics rather than the reverse (which works well, too, of course). Doing so would be less reasonable, for example, for those students who aspire to doctoral study in theoretical mathematics, as this is a highly specialized discipline that directly builds upon the depth of their previous mathematical training. We posit that an assumption that is often unexamined is that biostatistics is a highly specialized and mathematically intensive discipline rather like theoretical mathematics: this assumption holds true for a minority of graduate programs (whose admission criteria should reflect this) but not for most graduate programs and not for most practitioners.

As a practical matter, one recommendation we can provide to others is that implementing this sort of change in admission requirements requires proper framing. In particular, the goal of these changes was not to "admit students who are weaker at math", but instead to "enhance consistency between what we teach and who we admit". For our program, who we admit requires, at a minimum, acceptable background in each of the ABCs of biostatistics, with mathematics being part of the analytic competency. Once they have entered the pool of those with acceptable qualifications, applicants can distinguish themselves along multiple dimensions. To those who might have worried that requirements are being weakened, we clarified that (1) we continue to seek strong students, now with the criteria for what counts as "strong" being broadened; and (2) every admitted student has sufficient background in mathematics to succeed. Of course, we remain happy to admit students whose distinctive strength falls within the domain of mathematics and whose exposure extends far beyond the minimum.

CONCLUSION

In conclusion, we believe that removing unnecessary mathematical barriers and ensuring alignment between our mathematics requirements, curriculum content, and admission requirements is educationally sound and can also help increase access, reduce challenges, and support success for aspiring or emerging biostatisticians from diverse or multifaceted backgrounds. It is one way to contribute to a more inclusive and equitable learning environment in our STEM discipline. We believe similar programs might benefit from performing this type of analysis and reflection.

7 Appendix 1: Admission criteria for mathematics In

Admission requirement	Justification
Calculus	
Good grades in math-related courses	Some exposure to math is a surrogate for a generic ability to define and solve statistical problems using math

Facility with basic mathematical concepts such as functions	Used as a building block for what follows
Facility with integrals and derivatives for one variable	Used for working with statistical distributions -- for example, for transforming probability density functions to cumulative distribution functions
Facility with integration in 2 variables strongly preferred	Used for working with marginal and conditional distributions
Ability to manipulate functions (e.g., to find maxima and minima)	Used in working with likelihood functions, among others
Linear algebra	
Ability to manipulate matrices (e.g., add, invert, transpose)	Used as a building block for what follows
State models in matrix language	Used to succinctly describe statistical models
Project high-dimensional data into lower dimensions (e.g., eigenvalues, eigenvectors)	Used in multi-predictor models

8 Appendix 2: Illustration of Functional Mastery of Calculus

One of our admission criteria pertains to mastery of functions. To illustrate what is intended, consider the following question:

"Is $\exp\{-2t\}$, $t > 0$, a function of t ? How would you find its maximum value? Please discuss "why" in addition to "how".

A possible answer is as follows:

- A function is a rule, which takes inputs and uniquely assigns values for its outputs. Here, the inputs are the positive numbers (i.e., " $t > 0$ "). For each value of t , the output is $f(t) = \exp\{-2t\}$. This is a special case where the usual calculus procedure for finding a maximum of a function, which begins by finding the derivative of that function and setting it equal to 0, doesn't work.
- To see why the usual approach doesn't work, this derivative is $(-2) \cdot \exp\{-2t\}$. According to the rules of exponents, $\exp\{-2t\} = 1 / \exp\{2t\}$, and there is no value of x for which $1/x$ equals 0.
- We can, nevertheless, proceed using logic. Simply plugging in values of t makes it apparent that as t increases $f(t)$ decreases. In essence, as t increases so does $\exp\{2t\}$ and so $1 / \exp\{2t\}$ decreases, as does $2 / \exp\{2t\}$. So, the maximum value of $f(t)$ corresponds to the smallest value of t within its range.
- Now, if the range was $t \geq 0$ rather than $t > 0$, we could plug $t=0$ into $f(t)$ and obtain the maximum value $f(t)=1$. As we allow t to decrease and approach 0, the value of $f(t)$ approaches 1 as closely as you like. Indeed, in the world of calculus, "approaches 1 as closely as you like" has the same interpretation as "is 1", and so the maximum value of $f(t)$ is 1.

Comment: To understand the principles behind the above solution, a working knowledge of the nature of functions suggests that you should start by plotting $f(t)$ on the y-axis and t on the x-axis and hope that this provides a clue about the location of the maximum. Plugging in a few values of t makes it clear that $f(t)$ decreases as t increases, and so the maximum must occur at " $t=0$ ". This conclusion can be checked using rules of exponents and quotients, which are basic

mathematical techniques. Finally, the fundamental calculus principle of a limit translates "approaches 1" to "equals 1". The solution only relies upon basic mathematical manipulations, but does require the ability to use of the construct of general mathematical thinking in order to set up the analytical approach. In other words, general mathematical thinking suggests drawing a graph and using the shape of that graph to discover the likely value of the maximum.

References

- [1]. Biggs, J. Enhancing teaching through constructive alignment. *High Educ* 32, 347–364 (1996). <https://doi.org/10.1007/BF00138871>
- [2]. Neely ML, Troy JD, Gschwind GT, Pomann G-M, Grambow SC, Samsa GP. Preorientation curriculum: an approach for preparing students with heterogeneous backgrounds for training in a Master of Biostatistics program. *Journal of Curriculum and Teaching*. Online Published April 21, 2022. DOI:10.5430/jct.v11n4p77.
- [3]. Pomann G-M, Boulware LE, Chan CC, Grambow SC, Hanlon AL, Neely ML, Peskoe SB, Samsa G, Troy JD, Yang LZ, Thomas SM. Experiential learning methods for biostatistics students: A model for embedding student interns in academic medical centers. *Stat. online* 09Sep2022. <https://doi.org/10.1002/sta4.506>.
- [4]. Samsa GP, LeBlanc TW, Locke SC, Troy JD, Pomann, GM. A model of cross-disciplinary communication for collaborative statisticians: implications for curriculum design. 2018. *Journal of Curriculum and Teaching*. Online August 3, 2018. DOI:10.5430/jct.v7n2p1.
- [5]. Samsa G. Using coding interviews as an organizational and evaluative framework for a graduate course in programming. *Journal of Curriculum and Teaching*, 2020, 9(3), 107-140.
- [6]. Samsa G. Evolution of a qualifying examination from a timed closed-book format to a collaborative take-home format: a case study and commentary. *Journal of Curriculum and Teaching*. 2021. 10(1). 47-55. Doi:10.5430/jct.v10n1p47.
- [7]. Troy JD, Neely ML, Grambow SC, Samsa GP. The Biomedical Research Pyramid: A model for the practice of biostatistics. *Journal of Curriculum and Teaching*, 2021, 10(1), 10-17.
- [8]. Troy JD, Granek J, Samsa GP, Pomann G-M, Updike S, Grambow SC, Neely ML. A course in biology and communication skills for master of biostatistics students. *Journal of Curriculum and Teaching*. April 21, 2022. DOI: <https://doi.org/10.5430/jct.v11n4p77>.
- [9]. Troy JD, McCormack K, Grambow SC, Pomann G-M, Samsa GP. Redesign of a first-year theory sequence in biostatistics. *Journal of Curriculum and Teaching*. 2022;11(8),1-12. Doi:10.5430/jct.v11n8p1.
- [10]. Troy JD, Pomann G-M, Neely ML, Grambow SC, Samsa GP. Are simulated coding interviews a fair and practical examination format for non-professional programmers enrolled in a master's degree program in biostatistics? *Journal of Curriculum and Teaching*. 2023, 11(15), DOI:10.5430/jct.v12n6p253.
- [11]. Brown BA, Henderson JB, Gray S, Donovan B, Sullivan S, Patterson A, Waggstaff W. From description to explanation: an empirical exploration of the African-American pipeline problem in STEM. *Journal of Research in Science Teaching*, 2016, 53(1), 146-177.
- [12]. Cannady MA, Greenwald E, Harris KN. Problematizing the STEM pipeline metaphor: is the STEM pipeline metaphor serving our students and the STEM workforce? *Science Education*, 2014, 98(3), 443-460, DOI:10.1102/sce.21108.
- [13]. Estrada M, Burnett M, Campbell AG, Campbell PB, Denetclaw WF, Gutierrez CG, Hurtado S, John GH, Matsui J, McGee RM, Okpodu CM, Robinson TJ, Summers MF, Werner-Washburne M, Zavala ME. Improving underrepresented minority student persistence in STEM. *CBE-Life Sciences Education*. September 1, 2016 15:es5 DOI:10.1010.1187/cbe.16-01-0038.