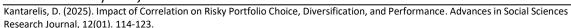
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Impact of Correlation on Risky Portfolio Choice, Diversification, and Performance

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ABSTRACT

Using Modern Portfolio Theory, applied on risky (stock) portfolios with real price data, it is shown that lower average portfolio correlation enables the investor to improve diversification and, consequently, experience lower portfolio risk as well as reach higher wealth indifference curves. Based on low and high correlation risky investments, results are calculated for Equally Weighted, Minimum Risk, Maximum Expected Return, and Maximum Sharpe Ratio portfolios. Long position performance is measured in terms of Expected Portfolio Return, Portfolio Standard Deviation, and Sharpe Ratio and, with the help of Monte Carlo simulation, it is shown that low correlation portfolios outperform high correlation portfolios. It is concluded that although low portfolio correlation is, undoubtedly, of paramount importance for diversification and portfolio choice, it is not a panacea: the investor must recognize that she needs both lower correlation and higher expected returns, must take into consideration the fact that the degree of correlation changes over time, and be aware of the fact that sometimes it may be beneficial to include in the portfolio positively correlated assets.

Keywords: Finance, Investments, Correlation, Risky Portfolio Choice, Modern Portfolio Theory, Efficient Frontier.

INTRODUCTION

Investing in multiple asset classes from different geographic regions, diverse sectors and exchange-traded funds, as well as regular monitoring of the portfolio to align with changing risk tolerance, investment goals, and changing market conditions are all necessary for everimproving portfolio diversification. Additionally, diversification calls for periodically aligning a portfolio to include negatively correlated assets. For example, if *Apple* stocks tend to gain whenever *Dell* stocks lose, including both in a portfolio will reduce risk. Alternatively, if *GM* and *Michelin* tend to prosper and suffer together, the decision to hold both stocks in a portfolio may not reduce portfolio risk. As more fully stated by Right Horizons (2025) in a section titled *Identifying Correlated and Uncorrelated Assets*,

"Effective diversification relies on identifying assets with low or negative correlations. This process involves analyzing historical price data and calculating correlation coefficients between different securities. For example, traditionally, stocks and bonds have shown low correlation, making them popular choices for diversified portfolios. However, correlations can change over time, necessitating ongoing analysis and portfolio adjustments."

Consider a hypothetical numerical example with two stocks in conjunction with Portfolio 1 (Figure 1) and Portfolio 2 (Figure 2): an investor owns 100 shares of stock a and 200 shares of stock b for a 12-period time (t) horizon. In both figures, columns 2 and 3 contain stock prices (Pa, Pb), columns 4 and 5 contain price rate of return (Ra, Rb), column 6 contains portfolio value (PV), and column 7 contains PV rate of return (Rpv.) In Figure 1, prices of a and b are positively correlated (Pearson correlation=0.371336) and the graph portrays Ra, Rb, and Rpv which move in tandem over time. In Figure 2, prices of a and b are negatively correlated (Pearson correlation=-0.63825) and the graph, like in Figure 2, portrays Ra, Rb, and Rpv; this time, the Rpv fluctuates less, because when A's returns rise, B's fall and vice-versa, causing the portfolio's performance to net-out most of these fluctuations. Obviously, if an investor could find two stocks like these, there would be considerable gain to diversifying among them.

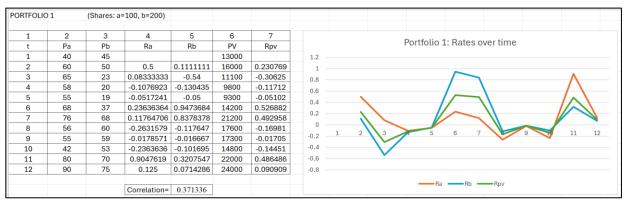


Figure 1: Portfolio with positively correlated stock prices

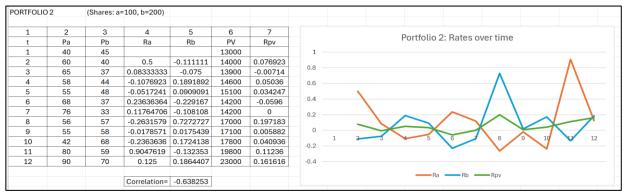


Figure 2: Portfolio with negatively correlated stock prices

More formally, following conventional risky portfolio methodology¹, summarized below by equations (1) to (4), the investor is assumed to invest \$1 in \mathbf{n} assets (where K=spending percentage or weight) by choosing K_1 , K_2 , ..., K_n to maximize (1) subject to (2), given (3) and (4):

Expected Wealth =
$$E\{W[EPR, PSD]\}$$
 (1)

Spending =
$$\sum_{i=1}^{n} K_i = 1$$
 (2)

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¹ Markowitz (1952).

Expected Portfolio Return = EPR =
$$\sum_{i=1}^{n} K_i E(X_i)$$
 (3)

Portfolio Standard Deviation or Risk = PSD =
$$\sigma_p = \left[\sum_{i=1}^n K_i^2 \sigma_i^2 + 2\sum_{i=1}^n \sum_{j=1, j \neq i}^n K_i K_j \sigma_i \sigma_j \rho_{ij}\right]^{\frac{1}{2}}$$
(4)

where, W = Wealth (which depends positively on EPR and negatively on PSD), and ρ = Correlation.

Because of the correlation coefficient in (4), the efficient combinations of risk and return or efficient frontier (EF), as the one in Figure 3 for a 2-stock portfolio, would shift to the left when returns of stocks x and y giving rise to EF₁, with correlation (ρ_1 =+1), are replaced by returns of stocks x and z giving rise to EF₂, with correlation (-1< ρ_2 <+1), where MRP=Minimum-Risk Portfolio. Zero risk is achieved when asset returns are perfectly negatively correlated with ρ =-1. Thus, leftward shifting of the EF, caused by lower correlation, enables the investor to reduce risk and reach a higher wealth indifference curve (IC), such as the one displayed (or any such positively sloped function), where e=equilibrium.

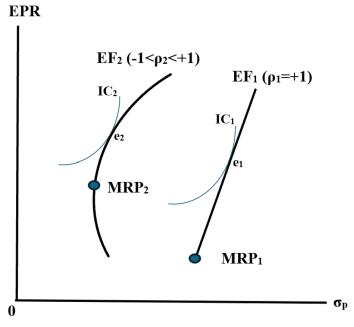


Figure 3: Efficient Frontier and Correlation

My objective in this paper is to utilize conventional methodology for a risky portfolio to examine the impact of portfolio correlation on diversification, return and risk and show that lower correlation enables the investor to experience lower risk and a higher indifference curve. I will rely on real-world price returns of risky assets assuming that such returns follow the normal distribution². I will start with 2-stock portfolios and utilize Pearson correlation,

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² The assumption that population stock returns follow the normal or the lognormal distribution has been made by many researchers, among others, Kendall & Hill (1953), Osborne (1959), Black & Scholes (1973), Sharpe (1964), and Phelan (1997). However, Li (2023), argues that these assumptions are made for mathematical ease and that in the real world such distributions are more likely to be more or less skewed instead of normal.

cognizant of its limitations in terms of linearity and sensitivity to the range of observations. Afterwards, I will experiment with 3-stock portfolios and utilize a measure that enables me to calculate the average correlation of assets in a portfolio.

TWO-STOCK PORTFOLIOS

Consider two risky long position investments, Investment 1 (consisting of stocks IBM and AAPL) and Investment 2 (consisting of stocks MSFT and AAPL), where IBM=International Business Machines, AAPL=Apple, and MSFT=Microsoft. Table 1, Panel A, displays the time horizon, sample observations of n=249 days, and the mean rate of return of the 3-Month Treasury Bill Rate (3MTBR) over the time horizon considered. Panels B and C display on the left summary measures; on the right, based on 21 arbitrarily chosen weights (K) and equations 3 and 4, they display the corresponding portfolio values for EPR, PSD and Sharpe Ratio (SR) [Sharpe (1996,1975,1992)] where SR ={[(EPR-(3MTBR Rate Mean)]|PSD}. The SR, calculated as such, is justified because it offers a valuable way to appraise the risk-adjusted return of a portfolio, allowing investors to compare different investment choices based on how much expected return they generate per unit of undertaken risk, making it an effective tool for evaluating performance of portfolio relative to its volatility. Highlighted in yellow are numerical results associated with portfolios "Equally Weighted", "Minimum PSD", "Maximum SR", and "Maximum EPR". (Approximately the same numerical values may be calculated using calculus or Excel's Solver tool.)

As we see, most of the low correlation efficient portfolios outperform the high correlation efficient portfolios: (a) all EPR and SR values in Panel B (except for portfolio 1) are greater than the EPR and SR values in Panel C; (b) most PSD values in Panel B (except for portfolios 18 to 21) are lower than the PSD values in Panel C. The efficient frontiers (EFs) in Panels B and C, graphed in Figure 4, reflect these results and clearly show that the investor (no matter how risk-tolerant) who decides to invest in the low correlation portfolios (blue) can reach higher wealth indifference curves.

Table 1: 2-Stock High vs. Low Correlation Portfolios(a)

D1 A								LLIUII		
	,									
anel A										
	Time Horizon (days)		9/13/2024							
	Sample size (n)	249								
	3MTBR Rate Average	0.000446								
	_									
anel B										
	Investment 1									
	Stocks	IBM	AAPL	Portfolio	KIBM	KAAPL	PSD	EPR	SR	
	Rate Average	0.001670	0.000992	1	0.00	1.00	0.013860	0.000992	0.039401	
	Rate Variance	0.000188	0.000192	2	0.05	0.95	0.013266	0.001026	0.043715	
	Rate Standard Deviation	0.013704	0.013860	3	0.10	0.90	0.012711	0.001060	0.048289	
	Rate Pearson Correlation	0.120228		4	0.15	0.85	0.012200	0.001094	0.053090	
				5	0.20	0.80	0.011737	0.001128	0.058069	
				6	0.25	0.75				
				7	0.30	0.70			0.068226	
				8	0.35		0.010703			
				9	0.40		0.010496			
				10	0.45		0.010365			
				11	0.50	0.50			0.085778	Minimum PSD=Equally Weighted
				12	0.55		0.010345			Minimum 152 Equally Weighten
				13	0.60		0.010455			
				14	0.65	0.35				
				15	0.70		0.010904			
				16	0.75	0.25			0.093830	Maximum SR
				17	0.73			0.001534		Maximum 51X
				18	0.80			0.001554		
				19	0.83			0.001308		
					0.90					
				20			0.013120			Mariana EDD
				21	1.00	0.00	0.013/04	0.0010/0	0.089276	Maximum EPR
1.C										
anel C										
anel C	Investment 2	Morri			V	W	non		an an	
anel C	Stocks	MSFT	AAPL	Portfolio		KAAPL	PSD	EPR	SR	
anel C	Stocks Rate Average	0.001159	0.000992	1	0.00	1.00	0.013860	0.000992	0.039401	
anel C	Stocks Rate Average Rate Variance	0.001159 0.000157	0.000992 0.000192	1 2	0.00	1.00 0.95	0.013860 0.013485	0.000992 0.001001	0.039401 0.041114	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3	0.00 0.05 0.10	1.00 0.95 0.90	0.013860 0.013485 0.013133	0.000992 0.001001 0.001009	0.039401 0.041114 0.042848	
anel C	Stocks Rate Average Rate Variance	0.001159 0.000157	0.000992 0.000192	1 2 3 4	0.00 0.05 0.10 0.15	1.00 0.95 0.90 0.85	0.013860 0.013485 0.013133 0.012807	0.000992 0.001001 0.001009 0.001017	0.039401 0.041114 0.042848 0.044590	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5	0.00 0.05 0.10 0.15 0.20	1.00 0.95 0.90 0.85 0.80	0.013860 0.013485 0.013133 0.012807 0.012508	0.000992 0.001001 0.001009 0.001017 0.001026	0.039401 0.041114 0.042848 0.044590 0.046322	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6	0.00 0.05 0.10 0.15 0.20 0.25	1.00 0.95 0.90 0.85 0.80 0.75	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7	0.00 0.05 0.10 0.15 0.20 0.25 0.30	1.00 0.95 0.90 0.85 0.80 0.75	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001042	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35	1.00 0.95 0.90 0.85 0.80 0.75 0.70	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001042 0.001051	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001042 0.001051 0.001059	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241 0.052708	
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625 0.011491	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001042 0.001051 0.001059 0.001067	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241 0.052708 0.054047	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.40 0.45	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625 0.011491 0.011395	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001042 0.001051 0.001059 0.001067	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241 0.052708 0.054047	Equally Weighted
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.40 0.45 0.50	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60 0.55	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625 0.011491 0.011395 0.011337	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001051 0.001059 0.001067 0.001076	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241 0.052708 0.054047 0.055234 0.056249	Equally Weighted
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11 12	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60 0.55 0.50 0.45	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625 0.011491 0.011395 0.011337 0.011319	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001059 0.001059 0.001067 0.001084 0.001092	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.051241 0.052708 0.054047 0.055234 0.056249	Equally Weighted Minimum PSD
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.40 0.45 0.50	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60 0.55	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625 0.011491 0.011395 0.011337 0.011319	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001059 0.001059 0.001067 0.001084 0.001092	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241 0.052708 0.054047 0.055234 0.056249	
anel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11 12	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60 0.55 0.50 0.45	0.013860 0.013485 0.013133 0.012807 0.012508 0.012209 0.012000 0.011795 0.011625 0.011491 0.011395 0.011319	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001059 0.001059 0.001067 0.001084 0.001092	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.051241 0.052708 0.054047 0.055234 0.056249	
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11 12 13	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.40 0.45 0.50 0.55 0.66	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60 0.55 0.50 0.45	0.013860 0.013485 0.013133 0.012807 0.012508 0.012299 0.012090 0.011795 0.011625 0.011491 0.011395 0.011337 0.011314 0.011341	0.000992 0.001001 0.001009 0.001017 0.001026 0.001042 0.001051 0.001059 0.001067 0.001084 0.001092 0.001099 0.001099 0.001099	0.039401 0.041114 0.042848 0.044590 0.046322 0.048023 0.049671 0.051241 0.052708 0.054047 0.055234 0.056249 0.057705	
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11 12 13 14	0.00 0.05 0.10 0.15 0.20 0.35 0.40 0.45 0.50 0.65 0.665	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.65 0.60 0.55 0.50 0.45 0.45 0.43 0.35	0.013860 0.013485 0.013133 0.012807 0.012209 0.012209 0.0112000 0.011795 0.011395 0.011397 0.011319 0.011341 0.011401 0.011500	0.000992 0.001001 0.001009 0.001012 0.001026 0.001034 0.001042 0.001051 0.001059 0.001067 0.001084 0.001092 0.001101 0.001101	0.039401 0.041114 0.042848 0.044590 0.044592 0.048023 0.049671 0.051241 0.052708 0.055034 0.056249 0.057075 0.058126 0.0587026	
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.55 0.60 0.65 0.70 0.75 0.75 0.75	1.00 0.95 0.90 0.85 0.80 0.75 0.60 0.65 0.55 0.55 0.45 0.40 0.35 0.30	0.013860 0.013485 0.013133 0.012807 0.012508 0.012239 0.012000 0.011795 0.011625 0.011491 0.011397 0.011319 0.011341 0.011341 0.011401 0.011500 0.011637	0.000992 0.001001 0.001009 0.001017 0.001034 0.001042 0.001059 0.001057 0.001067 0.001067 0.001084 0.001092 0.001101 0.0011019 0.0011019	0.039401 0.041114 0.042848 0.044590 0.044592 0.048023 0.049671 0.051241 0.0552708 0.055049 0.057075 0.057702 0.058126 0.058126	Minimum PSD
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 7 8 9 10 11 12 13 14 15 16	0.00 0.05 0.10 0.20 0.25 0.30 0.45 0.55 0.60 0.65 0.70 0.75	1.00 0.95 0.90 0.85 0.80 0.75 0.70 0.55 0.60 0.55 0.50 0.45 0.45 0.43 0.35 0.35 0.30 0.25 0.35	0.013860 0.013485 0.013133 0.012807 0.012807 0.012239 0.012200 0.01195 0.011491 0.011395 0.011341 0.011341 0.011500 0.011500 0.011650	0.000992 0.001001 0.001009 0.001017 0.001017 0.001034 0.001051 0.001059 0.001067 0.001084 0.001099 0.001101 0.001101 0.001109 0.001101 0.001109 0.001101	0.039401 0.041114 0.042848 0.044590 0.046592 0.048923 0.049671 0.051241 0.052708 0.054047 0.055234 0.057075 0.057702 0.058126 0.058349 0.058349 0.058349 0.058228	Minimum PSD
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 7 8 9 10 11 12 13 14 15 16 17 18 19	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.55 0.60 0.55 0.70 0.75 0.80	1.00 0.95 0.90 0.85 0.85 0.75 0.70 0.65 0.65 0.55 0.45 0.35 0.30 0.35 0.30 0.25 0.30 0.30 0.30 0.30 0.30 0.30 0.30 0.3	0.013860 0.013485 0.013485 0.012807 0.012508 0.012239 0.011209 0.011795 0.011491 0.011397 0.011319 0.011319 0.011500 0.011625	0.000992 0.001001 0.001009 0.001009 0.001017 0.001026 0.001051 0.001059 0.001067 0.001084 0.001092 0.001101 0.001101 0.001101 0.001117 0.001126	0.039401 0.041114 0.042848 0.042890 0.046322 0.048023 0.051241 0.051241 0.0552708 0.056249 0.057075 0.057702 0.058126 0.058349 0.058349 0.058339 0.058339 0.058379 0.058379	Minimum PSD
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 7 8 9 10 11 11 12 13 14 15 16 17 18 19 19 19 19 19 19 19 19 19 19 19 19 19	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.65 0.65 0.70 0.70 0.80 0.85	1.00 0.95 0.80 0.85 0.80 0.75 0.65 0.60 0.55 0.40 0.35 0.30 0.25 0.20 0.20 0.20	0.013860 0.013485 0.013485 0.012807 0.012508 0.012239 0.012000 0.01195 0.011491 0.011341 0.011341 0.011401 0.011637 0.011811 0.011811 0.012259	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001051 0.001059 0.001067 0.001084 0.001092 0.001101 0.001109 0.001109 0.001114 0.001134	0.039401 0.041114 0.042848 0.042849 0.046322 0.048023 0.052708 0.052708 0.055234 0.057705 0.057702 0.058349 0.058349 0.058379 0.058228 0.057913 0.057913	Minimum PSD Maximum SR
Panel C	Stocks Rate Average Rate Variance Rate Standard Deviaition	0.001159 0.000157 0.012532	0.000992 0.000192	1 2 3 4 5 6 7 7 8 9 10 11 12 13 14 15 16 17 18 19	0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.55 0.60 0.55 0.70 0.75 0.80	1.00 0.95 0.80 0.85 0.80 0.75 0.65 0.60 0.55 0.40 0.35 0.30 0.25 0.20 0.20 0.20	0.013860 0.013485 0.013485 0.012807 0.012508 0.012239 0.012000 0.01195 0.011491 0.011341 0.011341 0.011401 0.011637 0.011811 0.011811 0.012259	0.000992 0.001001 0.001009 0.001017 0.001026 0.001034 0.001051 0.001059 0.001067 0.001084 0.001092 0.001101 0.001109 0.001109 0.001114 0.001134	0.039401 0.041114 0.042848 0.042849 0.046322 0.048023 0.052708 0.052708 0.055234 0.057705 0.057702 0.058349 0.058349 0.058379 0.058228 0.057913 0.057913	Minimum PSD

⁽a) Stock prices and 3-Month Treasury Bill (3MTBR) data were made available by the Wall Street Journal: Stock Prices < https://www.wsj.com/market-data/quotes/SYMBOL>3MTBR < https://www.wsj.com/market-data/quotes/bond/BX/TMUBMUSD03M/historical-prices>

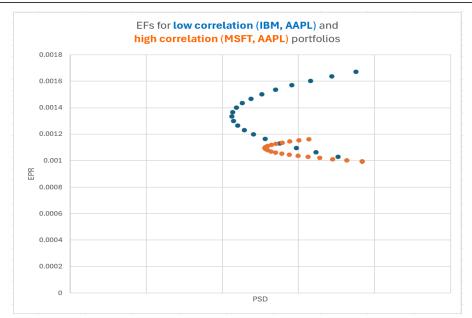


Figure 4: Efficient Frontiers (EFs) for Low and High Correlation 2-Stock Portfolios

THREE-STOCK PORTFOLIOS

According to Tierens et al. (2004), for a portfolio in which the number of assets exceeds two, *average* correlation may be justified as a useful measure. Nematrian.com (2025), credits Tierens et al. and clarifies that

"Investors are interested in the average correlation between stocks because it: (a) has a potential impact on their ability to add alpha; and (b) affects the level of portfolio risk they might be running. How might we best estimate and measure 'cross-stock' correlation? By 'best' we mean a suitable combination involving both (i) ease of computation and (ii) relevance to portfolio construction/risk analysis."

Nematrian.com adds that, intuitively, because "the volatility of a portfolio is typically lower than the weighted average volatility of the underlying constituents because stocks are less than perfectly correlated" the investor may define average portfolio correlation as follows:

PC = Average Portfolio Correlation =
$$\rho_a = \left(\sigma_p \middle| \sum_{i=1}^n k_i \sigma_i\right)^2 = \frac{\sum_{i=1}^n k_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1, j \neq i}^n k_i k_j \sigma_i \sigma_j \rho_{ij}}{\sum_{i=1}^n k_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1, j \neq i}^n k_i k_j \sigma_i \sigma_j}$$
(5)

To apply the above concepts with real world data, I compare two long position investments, each one of which consists of three stocks and various portfolios that may be formed out of them³. In turn, via Monte Carlo Simulation, I derive 10,000 random portfolios and I calculate the EPR, PSD, and PC values that correspond to functions (3), (4) and (5) above, which I then use to graph efficient frontiers.

119

³ The choice of two risky investments is made for simplicity purposes. Application of the concepts to portfolios that include more blends of investments from various asset classes, such as, among others, stocks, bonds, and commodities will infuse more complexity but, qualitatively, I believe, similar results.

Tabel 2, summarizes performance results of 10,000 efficient portfolios based on two investments each one consisting of three risky assets: Investment 1 (stocks MSFT, AAPL, LLY) and Investment 2 (stocks MSFT, AAPL, CAT) where, MSFT=Microsoft, AAPL=Apple, LLY=Eli Lilly, and CAT=Caterpillar.

Panel A reports time horizon dates, the corresponding sample size in days (n=207), and the average rate of the 3-Month Treasury Bill calculated over the time horizon days. It also enlists all applying acronyms.

Panels B and C display numerical results in eight blocks. Based on rates of return, Block 1 reports summary measures and Block 2 the Variance-Covariance matrix.

Blocks 3 to 6 report optimum values (calculated via Excel's Solver tool) for various portfolios: equally weighted, minimum PSD, maximum EPR, and maximum SR.

Block 7 uses Excel's RANDARRAY procedure to generate random spending percentages (weights) and, for one possible set of weights, reports the corresponding values for EPR (equation 3), PSD (equation 4), SR (Sharpe Ratio), and PC (equation 5.) Block 8 reports results of some iterations out of the 10,000 portfolios based on random weights generated via the RANDARRAY procedure and the corresponding values for EPR, PSD, SR, PC, as well as the average value of PC⁴.

As numbers in red show, Investment 1 portfolios in Panel B, with average PC=0.588, outperform Investment 2 portfolios in Panel C, with average PC=0.602. In Block 5, naturally, higher PSD in Panel B is justified compared to PSD in Panel C (higher EPR on the same efficient frontier requires higher PSD.) In Block 6 the results are identical in both panels because the best portfolio requires investing only in one and the same stock.

Based on the 10,000 random portfolios in Table 2 (Panels B and C, Block 8), Figure 5 portrays the corresponding low correlation Efficient Frontier (Panel B, Block 8, blue) along with the high correlation Efficient Frontier (Panel C, Block 8, orange.) Figure 5 reflects the results in Table 2 and clearly shows, as in the 2-Stock Portfolios case, that the investor (no matter how risk-tolerant) who decides to invest in the low correlation portfolios can reach higher wealth indifference curves.

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⁴ A Monte Carlo simulation, with constantly repeating random samples, does not have to be limited to a certain number of trials. To examine the sensitivity of results to the chosen number of trials, the simulation was run many times based on different numbers of trials such as 100, 1000, 5000, etc. Repetitive results based on trials ≥ 10,000 generated approximately the same values, hence my choice.

Table 2: 3-Stock High vs. Low Correlation Portfolios(b)

	_										
anel A											
		Time Horizon (da		11/3/2023			Definitions:				
		Sample size (n)	207				EPR=Expected Portfolio Re	turn			
		Rf Rate Average	0.000800155				PSD=Portfolio Risk				
							SR=Sharpe Ratio				
							PC=Portfolio Correlation				
							Rf=3MTBR=3-Month Trea	sury Bill Rate			
		,						1			
Panel B:	Low Average Correlation					Panel C: 1	High Average Correaltion				
Block 1	Investment 1					Block 1	Investment 2				
	Stocks	MSFT	AAPL	LLY			Stocks	MSFT	AAPL	CAT	
	Rate Average	0.00226116	0.00156079	0.00250235			Rate Average	0.00226116	0.00156079	0.00005366	
	Rate Variance	0.00026570	0.00016883	0.00033159			Rate Variance	0.00026570	0.00016883	0.00034311	
	Rate Standard Deviation	0.01630035	0.01299345	0.01820973			Rate Standard Deviation	0.01630035	0.01299345	0.01852329	
Block 2		VARCOVAR	Σ=XTX/(n-1)			Block 2			Σ=XTX/(n-1)		
		MSFT	AAPL	LLY				MSFT	AAPL	CAT	
	MSFT	0.00026570	0.00011934	0.00003583			MSFT	0.00026570	0.00011934	0.00003527	
	AAPL	0.00011934		0.00004284			AAPL	0.00011934	0.00016883	0.00006341	
	LLY	0.00003583	0.00004284	0.00033159			CAT	0.00003527	0.00006341	0.00034311	
Block 3	Equally Weighted	MSFT	AAPL	LLY		Block 3	Equally Weighted	MSFT	AAPL	CAT	
	Weights	0.33333333	0.33333333	0.33333333	1		Weights	0.33333333	0.33333333	0.33333333	
	EPR	0.00210810					EPR	0.00129187			
	PSD	0.01136341					PSD	0.01161268			
	SR	0.11510152					SR	0.04234306			
Block 4	Minimum PSD	MSFT	AAPL	LLY		Block 4	Minimum PSD	MSFT	AAPL	CAT	
	Weights	0.09043218	0.79119604	0.11837178	1		Weights	0.09043218	0.79119604	0.11837178	
	EPR	0.00173558					EPR	0.00144573			
	PSD	0.01176326					PSD	0.01193217			
	SR	0.07952107					SR	0.05410343			
Block 5	Maximum EPR	MSFT	AAPL	LLY		Block 5	Maximum EPR	MSFT	AAPL	CAT	
	Weights	0.00000000	0.00000000	1.00000000	1		Weights	1.00000000	0.00000000	0.00000000	
	EPR	0.00250235					EPR	0.00226116			
	PSD	0.01820973					PSD	0.01630035			
	SR	0.09347700					SR	0.08963045			
Block 6	Maximum SR	MSFT	AAPL	LLY		Block 6	Maximum SR	MSFT	AAPL	CAT	
	Weights	1.00000003	0.00000000	0.00000000	1		Weights	1.00000003	0.00000000	0.00000000	
	EPR	0.00226116					EPR	0.00226116			
	PSD	0.01630035					PSD	0.01630035			
	SR	0.08963045					SR	0.08963045			
Block 7	1/10000 Portfolios	MSFT	AAPL	LLY		Block 7	1/10000 Portfolios	MSFT	AAPL	CAT	
	RANDARRAY	0.99570691	0.86868469	0.97951172	2.8439		RANDARRAY	0.95482306	0.65521871	0.99044427	2.600
	Weights	0.35011982	0.30545507	0.34442511	1		Weights	0.36717100	0.25196009	0.38086891	
	EPR	0.00213030					EPR	0.00124393			
	PSD	0.01143856					PSD	0.01185018			
	SR	0.11628610					SR	0.03744859			
	PC=Portfolio Correlation	0.51444247					PC=Portfolio Correlation	0.52764314			
Block 8	Portfolio	PSD	EPR	PC		Block 8	Portfolio	PSD	EPR	PC	
	1	0.01270542	0.00234206	0.63282200			1	0.01233671	0.00088959	0.76810169	
	2	0.01153782	0.00194892	0.61832821			2	0.01287018	0.00143092	0.77012607	
	3	0.01157398		0.55337544			3	0.01158709	0.00133546	0.66484373	
		0.01153571	0.00212063	0.53310264			10000	0.01243728	0.00088571	0.58464658	

⁽b) Stock prices and 3-Month Treasury Bill (3MTBR) data were made available by the Wall Street Journal: Stock Prices < https://www.wsj.com/market-data/quotes/SYMBOL>3MTBR < https://www.wsj.com/market-data/quotes/bond/BX/TMUBMUSD03M/historical-prices>

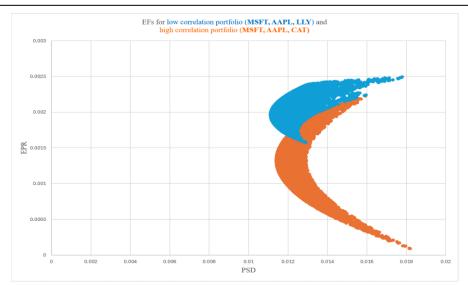


Figure 5: Efficient Frontiers (EFs) for Low and High Correlation 3-Stock Portfolios

SUMMARY & CONCLUSION

A clear message emerges from the above analysis based on risky investments: the lower the average portfolio correlation the more to a *northwest* location (in PSD, EPR space) the Efficient Forntier moves where the investor enjoys better diversification and, consequently, lower portfolio risk as well as a higher wealth indifference curve.

The rational investor ought to choose the low average correlation portfolio over the high average correlation portfolio but, correlation is not a magic bullet for it does not tell the whole story. As eloquently explained by Carlson (2014) and Roth (2025),

- a. The investor must recognize that she needs both lower correlation and higher expected returns. Inclusion of assets whose price fluctuations are partially or totally countered by other assets in a portfolio, may generate lower gains; it will be as if the investor simultaneously presses the gas pedal and the brake pedal for zero gains. Thus, inclusion of assets in a portfolio that cause average portfolio correlation to decrease will be beneficial but not at the expense of expected returns.
- b. Additionally, the investor must take into consideration the fact that the degree of correlation changes over time. There are no guarantees that negative (or other) correlations will persist. According to Carlson (2014),
 - "Investment environments are never quite the same across time. Interest rates, economic growth, industry leadership, inflation, innovation and a host of other factors are continuously changing as time marches on. It's impossible to extract perfect relationships in the movement of the markets strictly using past correlation data."
- c. Moreover, the investor has to be aware of the fact that sometimes it may be beneficial to include in the portfolio positively correlated assets. In the words of Roth (2025), "highly correlated assets may have very different returns, even if the high correlations continue." For example, suppose assets A and B are highly correlated. During the period 2000 to 2015, A gained 60% and B 20%. During the period 2016 to 2024, A gained 80%

and B 200%. Therefore, the investor will gain more if she includes both A and B assets in her portfolio even though they are positively correlated.

Concluding, as it was mentioned above, the choice to consider risky investments for simplicity purposes, was based on the *belief* that portfolios inclusive of more asset classes would not generate qualitatively different results. Belief though does not mean certainty. In the future, we ought to apply the proposed concepts to portfolios that include more blends of investments from various asset classes, such as, among others, stocks, bonds, commodities and real estate, which, with the aid of AI algorithms, will infuse more pragmatism by reducing undesirable complexity so that we detect with certainty the impact of portfolio correlation.

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