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An Improved the Prediction Accuracy of the Nonlinear Grey Bernoulli Model by Fourier Series and Its Application in Container Throughput Forecasting in Danang Port

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ABSTRACT

In order to improve the prediction accuracy of Nonlinear Grey Bernoulli Model NGBM (1,1), this study using Fourier series to modify their residual error of this model. To verify the effectiveness of the proposed approach, the annual water consumption in Wuhan from 2005 to 2012 is used for the modeling to forecast the annual water consumption demand from 2013 to May 2020, and the forecasting results proved that the Fourier- NGBM (1,1) is a better than the among forecasting model used in this situation. Furthermore, this proposed approach is applied the real case in forecasting the Container Throughput Forecasting in Danang Port. The empirical results show that the proposed model will get a higher accuracy performance with the lowest MAPE =1.93%. This result is not only show the effectiveness of proposed model but also offers valuable insights for Danang policymakers in orientation and planning management agency so as to boost the development of upcoming port activities.

Keywords: Nonlinear Grey Bernoulli model, Fourier series, Forecasting, Accuracy, Container, Danang Port.

INTRODUCTION

Grey forecasting is one of main part of Grey system theory, an effective method for modeling and forecasting small sample time series. In the early 1980s, Deng [1, 2] proposed the grey model GM (1,1) based on control theory, which is the core model used in the grey forecasting model. This model utilizes an operator obtained by first -order accumulation to operate on the *non-negative* original sequence. It demonstrates the approximate exponential growth laws and achieves short-term forecasting accuracy. With Its advantages in dealing with uncertain information and using as few as four data points [3,5], The GM (1,1) has been validated and widely used in various fields such as tourism [6, 7], transportation [8- 10], financial and economic [11- 13], integrated circuit industry [14-17], energy industry [18-20] etc...

In the recent years, there are many scholars propose new procedures with different ways to improve the precision accuracy of GM (1,1) model. For instant, Lin et al. [21] and Wang et al. [22] used different methods to calculate new background values to improve the background values. Hsu [17] and Wang et al. [23] used different methods to modified internal parameter estimation like development coefficient and grey input coefficient. Some scholars have established GM (1,1) model with residuals modification like Hsu [15] and Wang et al. [24]. In addition, many hybrid models based on GM (1,1) were proposed. These include the grey econometric model [25], the grey Markov model [27, 28], and the grey fuzzy model [21], etc. Despite its improvement in prediction accuracy, the prediction accuracy of the GM (1,1) model is always monotonic. As a result, GM (1,1) model may not be always satisfactory. The recently developed nonlinear grey Bernoulli model NGBM (1,1) is a new grey forecasting model [28]. It has a power exponent *n* that can effectively manifest the nonlinear characteristics of real systems and flexibility determines the shape of the model' curve. Unlike GM (1,1) and the grey Verhulst model which rely on a constant number such as 0 or 2, the NGBM (1,1) does not require such a number (excluding 1).

Therefore, forecasting of the fluctuation sequence can be performed by the fluctuation features as long as the power exponent and structural parameters in the model are known. The NGBM (1,1) was successfully used to simulate and forecast the values of the annual unemployment rates of ten selected countries and foreign exchange rates of Taiwan's major trading partners [29, 30]. This success indicates that the NGBM (1,1) significantly improves the accuracy of the simulation and forecasting predictions of the original GM (1,1). Zhou et al. [30] selected the value of *n* by using a particle swarm optimization algorithm and used the model to forecast the power load of the Hubei electric power network. Hsu [16] used the genetic algorithm to optimize parameters of the NGBM (1,1) and applied it to forecast the economic trends in the integrated circuit industries in Taiwan. Chen et al. [31] proposed a Nash NGBM (1,1) in which an interpolated coefficient in the background value is introduced into the NGBM (1,1) and the parameters solved for based on the Nash equilibrium concept. This strengthens the adaptability of the model towards the original data and eventually improves the accuracy of the model. Later, Wang, et al. [32] proposed optimized NGBM (1,1) model to forecast the qualified discharge rate of the industrial waste water in 31 administrative areas in China by improved background interpolation value *p* and exponential value *n* is put forward in an NGBM (1,1). Pao et al. [33] forecasted $CO2$ emissions, energy consumption, and economic growth in China based on ARIMA and NGBM (1,1). Performance evaluation results showed that the NGBM (1,1) can be used safely for future projection of these indicators in clean energy economy.

All these improvements focus on the model parameters and the background value. In fact, the initial condition is also an important factor determining the grey modeling accuracy. This is because the initial condition is a part of the predictive function. The current paper aims to develop an approach to increase the predictive precision of the NGBM (1,1) by modifying the residual error obtained from NGBM (1,1) with Fourier series. Numerical example and practical application shows that the proposed Fourier-NGBM (1,1) model has higher performance on models prediction. The remainder of this paper is organized as follows. A brief introduction to the original NGBM (1,1) and the method of optimization of the modified residual error by Fourier series are given in Section 2. Section 3 proves the effectiveness of using the proposed Fourier-NGBM (1,1) by comparison with the Coupling model of Grey system and Multivariate

Linear Regression for water consumption forecasting in He et al. [34]. Section 4 illustrate the application of Fourier-NGBM (1,1) in forecast the Container Throughput in Danang Port, Finally, the paper concludes with some comments in Section 5.

METHODOLOGY

A Brief Introduction to the Nonlinear-Grey Bernoulli Model "NGBM (1,1)"

NGBM (1,1) is a first-order single-variable grey Bernoulli model with an interpolated coefficient in the background value [31]. For predications involving nonlinear small sample time series, its performance is better than that of the original grey forecasting models and NGBM (1,1) itself. The procedures involved for using the NGBM (1,1) can be summarized as follows:

Step1: Let raw matrix $X^{(0)}$ stands for the non-negative original historical time series data

$$X^{(0)} = \left\{x^{(0)}(t_i)\right\},\ i = 1,2,\ldots,m \tag{1}$$

Where $x^{(0)}(t_i)$ corresponds to the system output at time t_i, and m is the total number of modeling data.

Step 2: Construct $X^{(1)}$ by one time accumulated generating operation (1-AGO), which is

$$X^{(1)} = \left\{x^{(1)}(t_i)\right\},\ i = 1,2,\ldots,m \tag{2}$$

where

$$x^{1}(t_k) = \sum_{i=1}^{k} x^{(0)}(t_i), k = 1,2,\ldots,m \tag{3}$$

Step 3: $X^{(1)}$ is a monotonic increasing sequence which is modeled by the Bernoulli differential equation:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b\left[X^{(1)}\right]^{n} \tag{4}$$

Where the parameter "*a*" is called the developing coefficient and "*b*" is the named the grey input and "*n*" is any real number (excluding *n*=1).

Step 4: In order to estimate the parameter "a" and "b", Eq. (4) is approximated as:

$$\frac{\Delta X^{(1)}(t_k)}{dt_k} + aX^{(1)}(t_k) = b\left[X^{(1)}(t_k)\right]^{n} \tag{5}$$

Where

$$\Delta X^{(1)}(t_k) = x^{(1)}(t_k) - x^{(1)}(t_{k-1}) = x^{(0)}(t_k) \tag{6}$$

$$\Delta t_k = t_k - t_{k-1} \tag{7}$$

If the sampling time interval is units, then let $\Delta t_k = 1$, Using

$$z^{(1)}(t_k) = px^{(1)}(t_k) + (1-p)x^{(1)}(t_{k-1}),\ k = 2,3,..,n \tag{8}$$

To replace $X^{(1)}(t_k)$ in Eq. (5), we obtain

$$x^{(0)}(t_k) + az^{(1)}(t_k) = b\left[z^{(1)}(t_k)\right]^n,\ k = 2,3,\dots,m \tag{9}$$

Where $z^{(1)}(t_k)$ in Eq. (8) is termed background value, and *p* is production coefficient of the background value in the range of (0, 1), which is traditionally set to 0.5.

Step 5: From the Eq. (9), the value of parameter "*a*" and "*b*" can be estimated using least- square method. That is

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_m \tag{10}$$

Where

$$B = \begin{bmatrix} -z^{(1)}(t_2) & (z^{(1)}(t_2))^n \\ -z^{(1)}(t_3) & (z^{(1)}(t_3))^n \\ \dots\dots & \dots\dots \\ -z^{(1)}(t_m) & (z^{(1)}(t_m))^n \end{bmatrix} \tag{11}$$

And

$$Y_m = \left[x^{(0)}(t_2), x^{(0)}(t_3), \dots, x^{(0)}(t_m))\right]^T \tag{12}$$

Step 6: The solution of Eq. (4) can be obtained after the parameter "*a*" and "*b*" have been estimated. That is

$$\hat{x}^{(1)}(t_k) = \left[\left(x^{(0)}(t_1)^{(1-n)} - \frac{b}{a}\right)e^{-a(1-n)(t_k - t_1)} + \frac{b}{a}\right]^{\frac{1}{1-n}}, n \neq 1,\ k = 1,2,3,\dots \tag{13}$$

Step 7: Applying inverse accumulated generating operation (IAGO) to $\hat{x}^{(1)}(t_k)$, the predicted datum of $x^{(0)}(t_k)$ can be estimated as:

$$\begin{cases} \hat{x}^{(0)}(t_1) = x^{(0)}(t_1) & (14) \\ \hat{x}^{(0)}(t_k) = \hat{x}^{(1)}(t_k) - \hat{x}^{(1)}(t_{k-1}) & (15) \end{cases}, \ k = 2,3,\ldots$$

Modifying the Residual Error of NGBM (1,1) by Fourier Series "Fourier- NGBM (1,1)"

In order to improve the prediction accuracy of Nonlinear Grey Bernoulli model, the Fourier series was used to modify the residual error of the NGBM (1,1). The overall procedure to obtain the modified model is as the followings [6]:

Let x is the original series of m entries and v is the predicted series (obtained from NGBM (1,1). Based on the predicted series v, a residual series named ε is defined as:

$$\varepsilon = \{\varepsilon(k)\},\ k = 2,3,\ldots m \quad (16)$$

Where

$$\varepsilon(k) = x(k) - v(k)\,,\ k = 2,3,\ldots m \quad (17)$$

According to the definition of the Fourier series, the residual sequence of NGBM (1,1) can be approximately expressed as:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_{(0)} + \sum_{i=1}^{Z}\left[a_i \cos\left(\frac{2\pi i}{m-1}(k)\right) + b_i \sin\left(\frac{2\pi i}{m-1}(k)\right)\right], k = 1,2,3,.,m \quad (18)$$

Where $Z = (\frac{m-1}{2}) - 1$ called the minimum deployment frequency of Fourier series [35] and only take integer number, therefore, the residual series is rewritten as:

$$\varepsilon = P.C \quad (19)$$

Where

$$P = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{m-1}\times 2\right) \sin\left(\frac{2\pi\times 1}{m-1}\times 2\right) & \ldots & \cos\left(\frac{2\pi\times Z}{m-1}\times 2\right) \sin\left(\frac{2\pi\times Z}{m-1}\times 2\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{m-1}\times 3\right) \sin\left(\frac{2\pi\times 1}{m-1}\times 3\right) & \ldots & \cos\left(\frac{2\pi\times Z}{m-1}\times 3\right) \sin\left(\frac{2\pi\times Z}{m-1}\times 3\right) \\ \ldots & \ldots & \ldots & \ldots \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{m-1}\times m\right) \sin\left(\frac{2\pi\times 1}{m-1}\times m\right) & \ldots. & \cos\left(\frac{2\pi\times Z}{m-1}\times m\right) \sin\left(\frac{2\pi\times Z}{m-1}\times m\right) \end{bmatrix} \quad (20)$$

And

$$C = [a_0, a_1, b_1, a_2, b_2, \ldots, a_Z, b_Z] \quad (21)$$

The parameter a_0, a_1, b_1, a_2, b_2... a_Z, b_Z are obtained by using the ordinary least squares method (OLS) which results in the equation of:

$$C = \left(P^T P\right)^{-1} P^T [\varepsilon]^T \tag{22}$$

Once the parameters are calculated, the modified residual series is then achieved based on the following expression:

$$\hat{\varepsilon}(k) = \frac{1}{2} a_{(0)} + \sum_{i=1}^{Z} \left[a_i \cos\left(\frac{2\pi i}{m-1}(k) \right) + b_i \sin\left(\frac{2\pi i}{m-1}(k) \right) \right] \tag{23}$$

From the predicted series v and $\hat{\varepsilon}$, the Fourier modified series $\hat{v}$ of series v is determined by:

$$\hat{v} = \{\hat{v}_1, \hat{v}_2, \hat{v}_3, \ldots., \hat{v}_k, \ldots, \hat{v}_m\} \tag{24}$$

Where

$$\hat{v} = \begin{cases} \hat{v}_1 = v_1 \\ \hat{v}_k = v_k + \hat{\varepsilon}_k \quad (k = 2,3,\ldots,m) \end{cases} \tag{25}$$

Evaluative Precision of Forecasting Models

In order to evaluate the forecast capability of the model, Means Absolute Percentage Error (MAPE) index was used to evaluate the performance and reliability of forecasting technique [35]. It is expressed as follows:

$$MAPE = \frac{1}{m} \sum_{k=2}^{m} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \tag{26}$$

Where $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ are actual and forecasting values in time period k, respectively, and n is the total number of predictions. Wang et al. [36] interprets the MAPE results as a method to judge the accuracy of forecasts, where more than 50% is an inaccurate forecast, 20%-50% is a reasonable forecast, 10%-20% is a good forecast, and less than 10% is an excellent forecast.

VALIDATION OF THE FOURIER- NGBM (1,1)

To show the effectiveness of the Fourier-NGBM (1,1), This paper the first take the numerical example and then compare the performance among Fourier-NGBM (1,1) and Coupling model of Grey system and Multivariate Linear Regression as well as original NBBM (1,1) for forecasting the annual water consumption demand in Wuhan [34]. Based on the historical data in He et al. [34], the annual water consumption demand from year 2005 to 2012 are used to forecast the data from 2013 - 2020, To calculate the parameters in NGBM (1,1) and Fourier-NGBM (1,1) forecasting models, computer software called Microsoft Excel is used. Beside a basic function in excel, Excel software also offers two useful functions named Mmult (array 1, array 2) to return the matrix product of two relevant arrays and Minverse (array) to return the inverse matrix. These two functions and Excel- solve software are of great help to find out and to optimize the values of parameters in forecasting models. For the sake of convenience, the detailed calculation and modeling process are omitted here. All the results of two models are shown in Table 1, Only the MAPE of these models compared with coupling model of Grey system and Multivariate Linear Regression are shown in Table 2.

Table1: Forecasting results from the NGBM (1,1) and Fourier-NGBM (1,1) models

Actual value		**Original NGBM (1,1) n= -0.11, p=0.5**		**Fourier-NGBM (1,1) n= -0.11, p=0.5**	
	$x^{(0)}(k)$	$\hat{x}^{(0)}(k)$	Residual error (%)	$\hat{x}^{(0)}(k)$	Residual error **(%)**
2005	367,204	367,204	0	367,204	**0**
2006	369,355	369,355	0	369,355	**0**
2007	363,883	364,331.6	0.1	363,883	**0**
2008	360,467	366,777.6	1.8	360,467	**0**
2009	377,944	372,695.6	1.4	377,944	**0**
2010	379,346	380,754.6	0.4	379,346	**0**
2011	397,345	390,359.2	1.8	397,345	**0**
2012	395,713	401,199.8	1.4	395,713	**0**
MAPE			0.8		**0**

Table 1 reveals that the Fourier-NGBM (1,1) with n = -0.11 and p = 0.5 has a higher accuracy than that of the Original NGBM (1,1) in term of the MAPE of the Fourier-NGBM (1,1) decreased from 0.8% to 0 %. This result also using compare with Coupling model of Grey system and Multivariate Linear Regression Model in [34] as well as GM (1,1). The overall MAPE indexes as well as the performance forecasting of these models was shown in table 2.

Table 2: Summary of evaluation indexes of model accuracy

Model	**MAPE (%)**	**Forecasting accuracy (%)**	**Performance**
GM (1,1)	1.163	98.837	Good
Coupling model of Grey system and Multivariate Linear Regression Model in [34]	1.156	98.84	Good
Original NGBM (1,1)	0.8	99.2	Excellent
Fourier - NGBM (1 ,1)	**0**	**100**	**Excellent**

Table 2 shows that the forecasting accuracy of the Fourier-NGBM (1,1) is the best among three forecasting models with 0 % of MAPE in forecasting the total annual water consumption Demand in Wuhan.

APPLICATION OF FOURIER-NGBM (1,1) IN THE CONTAINER VOLUME FORECASTING

Data

The volume of containers through Danang Port during the period from 2011 to 2023 was gathered from Danang Port Joint Stock Company. The numeric data as a whole is visualized by Figure 1 and measured by TEUs [1].

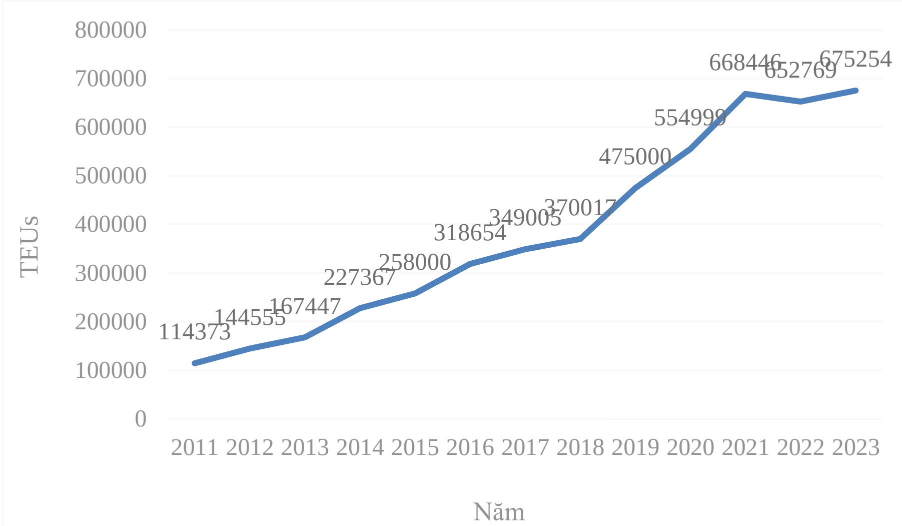

Figure 1: The volume of containers through Danang Port (Units: TEUs)

As can be clearly seen from Figure 1, there was a substantial increase in the volume of containers through Danang Port annually. This data reached 675,254 TEUs, increased more than 200.000 TEUs compared with 2011.

Tool and Functions

For calculation and stimulation of Grey forecasting model NGMB (1,1) and Fourier - NGBM (1,1), this research will use Microsoft Excel of Microsoft Corporation. This is a common software among users and multiple functions are integrated for calculation. To calculate the parameter value of these two models above, this research execute basic algorithms and 02 useful functions for matrix by matrix multiplication "Mmult (matrix 1, matrix 2)" and matrix inverse calculation "Minverse (matrix)". They are two fundamental calculations of in model parameter values. After executing these calculations by Microsoft Excel. Forecasting results of two models is shown below.

The Forecasted Performance of NGBM (1,1) Model

In combination with the data set gathered from 2013 to 2023 and algorithm of NGBM (1,1) forecasting model which is indicated in section 2.1, this research found out the parameter value a= -0.02 and b = 288.06, n=0.5. The NGBM (1,1) behavioral model for forecasting container volumes through Danang Port as follow: $\hat{x}^{(1)}(k) = (14729.62624 \times e^{0.01 \times k} - 14391.43547)^2$. All result are summarize as in Table 3.

Table 3: Forecasted and error values of NGBM (1,1) model

Year	Actual Value	Forecasted value by NGBM (1,1)	Error (%)
2011	114,373	-	-
2012	144,555	122,159.00	15.49
2013	167,447	167,951.94	0.30
2014	227,367	215,106.47	5.39
2015	258,000	263,654.50	7.49
2016	318,654	313,628.62	1.58
2017	349,005	365,062.15	4.60

2018	370,017	417,989.10	12.96
2019	475,000	472,444.24	0.54
2020	554,999	528,463.05	4.78
2021	668,446	586,081.82	12.32
2022	652,769	645,337.59	1.14
2023	675,254	706,268.21	4.59
MAPE			5.93%
Accuracy= (100-MAPE) (%)			94.07%
Evaluation			Excellence

The Forecasted Performance of Fourier-NGBM (1,1) Model

The residual series gained from NGBM (1, 1) In the session 4.3 is then modified with Fourier series, which results in the modified model FRMGM (1, 1), In as per the algorithm stated in section 2.2. The evaluation indexes of Fourier- NGBM (1,1) are summarized as in Table 4 and visualization of NGBM (1,1) and Fourier-NGBM (1,1) are shown in Figure 2

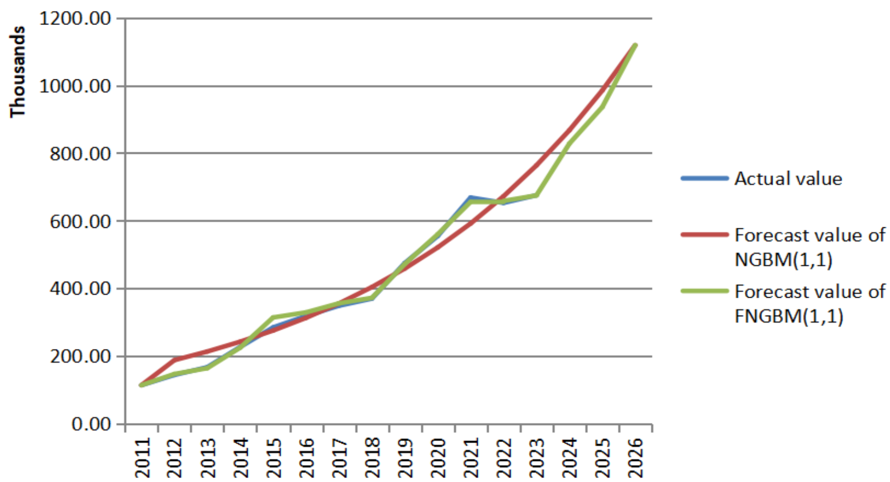

Figure 2: The visualization of NGBM (1,1) and Fourier- NGBM (1,1) of containers through Danang Port

Table 4: Forecasted and error values of Fourier- NGBM (1,1) model

Year	Actual Value	Forecasted value by Fourier- NGBM (1,1) model	error
2011	114,373	-	-
2012	144,555	164,696.00	1.90
2013	167,447	234,493.28	1.64
2014	227,367	321,356.81	3.13
2015	258,000	349,214.30	12.76
2016	318,654	368,991.98	9.59
2017	349,005	381,521.34	5.73
2018	370,017	472,249.00	3.11
2019	475,000	551,101.58	0.58
2020	554,999	640,544.96	0.70
2021	668,446	641,416.94	4.17
2022	652,769	660,289.40	1.74
2023	675,254	787,008.80	2.22

MAPE	3.94%
Accuracy= (100-MAPE) (%)	96.06%
Evaluation	Excellence

Research result in Figure 4 shows that result with MAPE co-efficient produced by Fourier-NGBM (1,1) model reaches 3.94% less than the result of NGBM (1,1) forecasting model. Otherwise, the fluctuation of MAPE co-efficient of values from 2013 to 2023 was within the range between 0.58% and 12.76%. Through this result, this research showed that the effectiveness of the Fourier- NGBM (1,1) forecasting model (with MAPE coefficient < 10%) is reliable.

The Volume of Container Throught Danang Port in the Year of 2025 and 2026

By comparing the accuracy between these two forecasting models aforementioned, this research propose the use of Fourier-NGBM (1,1) model to forecast the containers volume through Danang Port during the period betwen 2025 and 2026 because the accuracy of Fourier-NGBM (1,1) model is better than NGBM (1,1) forecasting model. Forecasting result is illustrated by Table 4.

Table 4: The container volumes through Danang Port during the period from 2025 to 2026

Year	Container volume throught Danang Port the year of 2025 and 2026 (Units: TEUs)
2025	830,053.50
2026	899,499.88

From the results of Figure 4, we can see that container volumes through Danang Port forecast continue to significantly grow during the next years. To be more detailed, containers volume through Danang Port will reach 830,053.50 TEUs in 2025 (38% increase compared with 2023) and this data is projected to hit the milestone of nearly 90 thousand TEUs in the year of 2026 . This will be an important data resources for orientation and planning management agency so as to boost the development of upcoming port activities.

CONCLUSION

In this paper, by using the Fourier series modifies the residual errors of NGBM (1,1). We propose an effectiveness NGBM (1,1) model termed as Fourier - NGBM (1,1). Through the simulation of the numerical example in He et al.' paper and its application in forecast the volume of container through Danang Port, this results displayed the Fourier- NGBM (1,1) is the better model in numerical cases with the value of MAPE is smallest. By the way, the simulation results in forecast the volume of container through Danang Port will be an important data resources for orientation and planning management agency so as to boost the development of upcoming port activities. In the future direction, this study will be modified with other ways to improve the accuracy of the NGBM (1,1) model and will be applied this model to expand on these findings and to forecast performance of different industries.

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